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The Transient response of heat exchangers

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The Transient Response of Heat Exchangers

by

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of the requirements for the degree of
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Abstract

The transient response of heat exchangers to a change in inlet temperature of one of the fluids is of much interest in industrial practice. Due to the complexity of the problem, no generally accepted solutions exist. This thesis presents an extensive survey of major solutions available in literature and identifies the ranges of parameters for which solutions are not available. A commercially available thermal network solver software package (Thermonet) will be used to model the transient response of heat exchangers. The software package will be verified using five existing solutions found in literature. The software package will be utilized to generate transient solutions for a counterflow heat exchanger covering a wide range of parameters useful in engineering practice. The results are presented in tabular form. Important parametric influences are discussed and effects of process and geometrical variables on the transient performance is evaluated.

Acknowledgments

I would like to express my sincere gratitude to Dr. Satish Kandlikar whose insight, encouragement, and guidance were crucial to the completion of this work. I would also like to thank my wife Hoimu for her love and support, and above all thanks to God, for without Him we can do nothing.

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May 12, 1995

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List of Symbols

A	heat transfer area
C	fluid heat capacity rate (equation 2.2)
c_p	specific heat at constant pressure
C	heat capacitance (equation 2.1)
C^*	dimensionless heat capacity rate (equation 2.10)
C_w^*	dimensionless wall heat capacitance (equation 2.13)
E	dimensionless quantity defined by Romie (equation 3.10 and 3.19)
h	heat transfer coefficient
L	heat exchanger length
M	mass of fluid or wall material in heat exchanger
NTU	number of heat transfer units(equations 2.9, 3.12 and 3.21)
q	heat transfer rate
R	dimensionless quantity defined by Romie (equation 3.11 and 3.20)
R^*	dimensionless thermal resistance ratio (equation 2.11)
T	temperature
$T(0)$	Steady state outlet temperature before transient input has been applied
$T(t)$	Outlet temperature at time t
$T(\infty)$	Steady state outlet temperature after transient input has been applied
T^*	dimensionless temperature (equation 2.7)
t	time
t^*	dimensionless time (equation 2.12)
t_d	dwell time, the amount of time required for a fluid particle to pass through the heat exchanger
t_d^*	dimensionless dwell time (equation 2.14)
$t_{d,min}$	dwell time of C_{min} fluid
U	overall heat transfer coefficient
V	dimensionless quantity defined by Romie (equation 3.13 for parallel flow, equation 3.22 for counterflow))
W	mass flow rate
X^*	dimensionless distance (equation 2.8)
x	position along flow direction

Greek

η_o	fin efficiency
ε_f^*	dimensionless temperature response (equations 2.16 and 2.17)
θ	dimensionless quantity defined by Romie (equations 3.9, 3.17 and 3.18)

Subscripts

h	hot fluid
c	cold fluid
w	heat exchanger wall
i	initial
min	minimum heat capacitance rate fluid
max	maximum heat capacitance rate fluid
1	stepped fluid
2	unstepped fluid

1. Introduction

A heat exchanger is said to be operating in the steady state when the inlet and outlet temperatures of the two fluids are constant over time. As one of the fluids experiences a change in its inlet temperature, the heat exchanger undergoes a transient excursion. When a heat exchanger is part of a complex system such as a process plant, power plant, or air conditioning system, the system designer often is required to determine the performance of the heat exchanger under transient operating conditions. This information may be required either for process control or for determining the influence of thermal stresses in the different components of the heat exchanger. Figure 1.1 shows a typical counterflow heat exchanger commonly used in engineering practice. Figure 1.2 shows steady state operating conditions before and after a step change in hot fluid inlet temperature for a typical counterflow heat exchanger.

There are three major types of transient inputs:

1) Step input, Figure 1.3. A sudden change in inlet temperature or flow rate to a new, constant value.

2) Frequency input, Figure 1.4. A periodically varying change in inlet temperature or flow rate.

3) Impulse input, Figure 1.5. A change in inlet temperature or flow rate of infinite amplitude but infinitesimal duration.

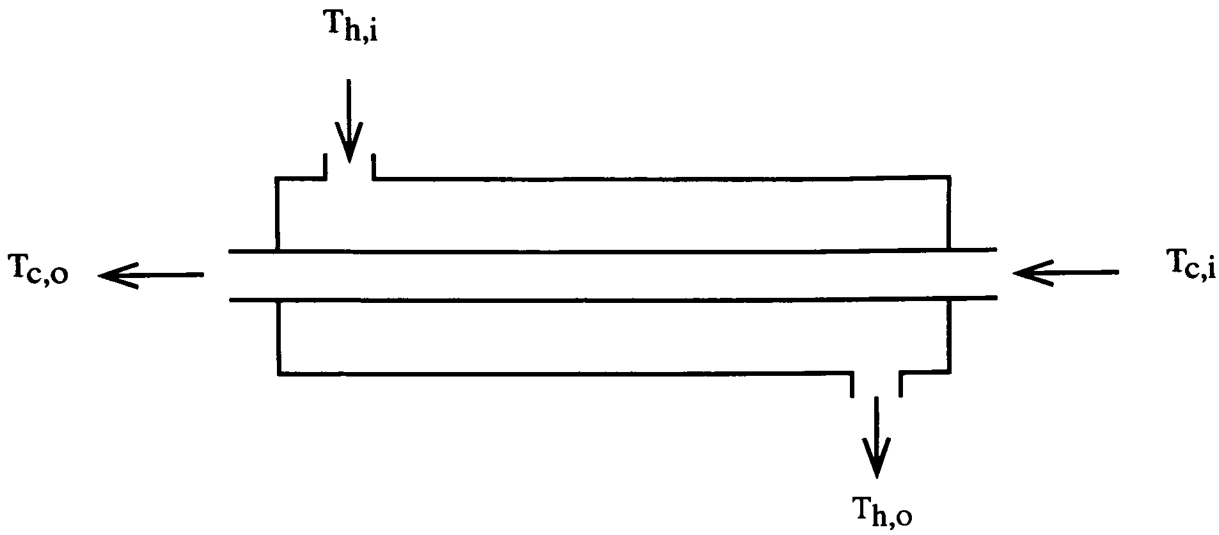


Figure 1.1 Typical counterflow heat exchanger

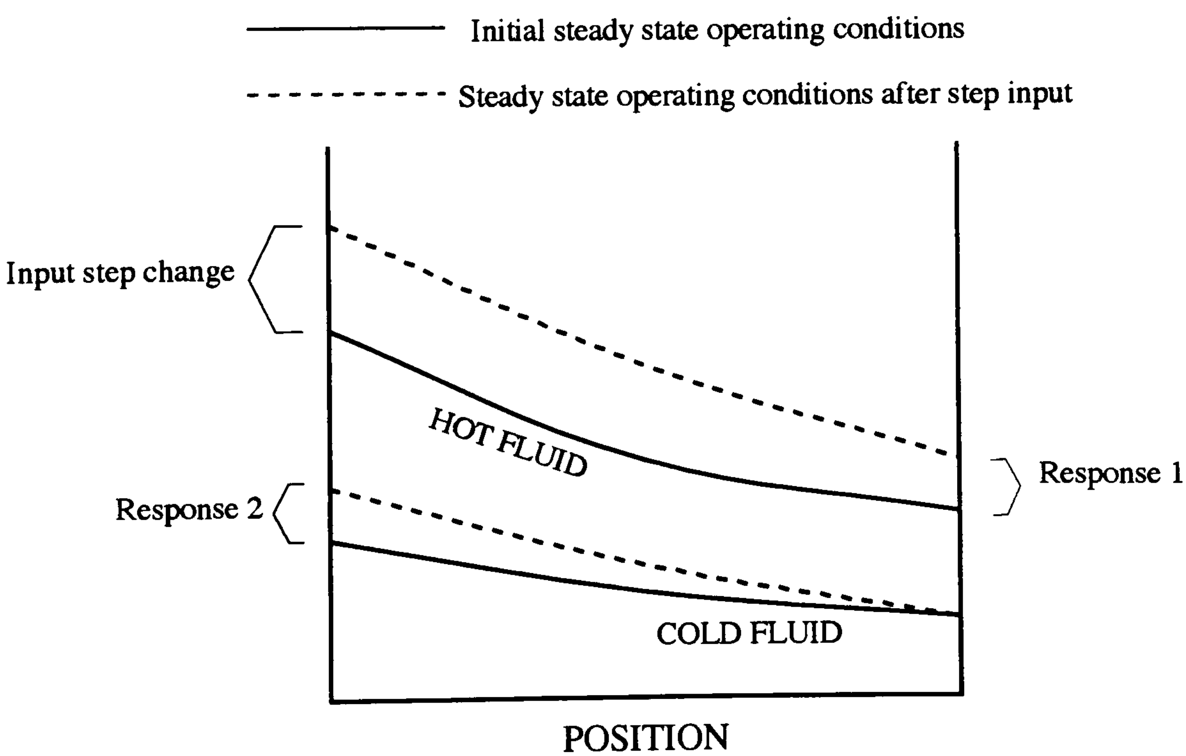


Figure 1.2 Steady state operating condidtions before and after a step change in hot fluid inlet temperature.

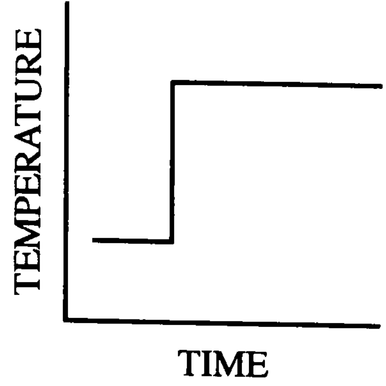


Figure 1.3 Step input

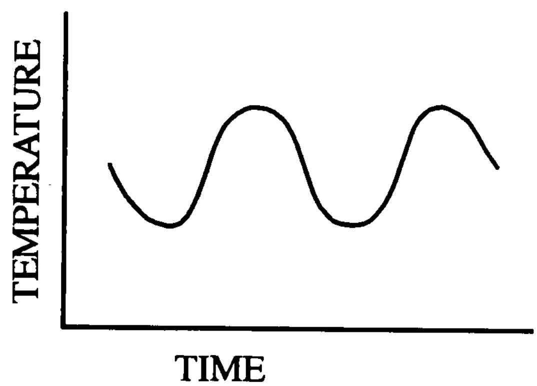


Figure 1.4 Frequency input

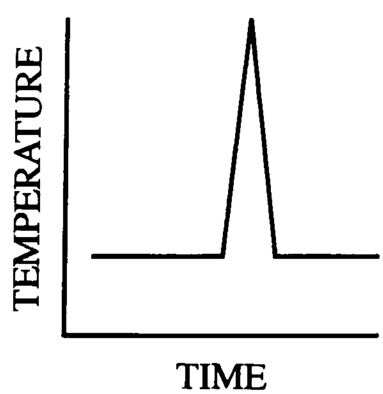


Figure 1.5 Impulse input

In many applications, the transient disturbances that occur in heat exchangers are approximated closely by a step function. For this reason most solutions to the transient problem are for step functions at the inlet. Therefore the focus of the present work is aimed at studying the transient response of heat exchangers subjected to a step inlet condition.

The solutions available for steady state heat exchanger analysis are readily available, well verified and generally convenient to utilize. The same cannot be said for transient heat exchanger analysis. As will be seen later, the transient problem is much more complex and no generally applicable solutions exist. Many authors have presented solutions in literature which are applicable over limited ranges of parameters. The most recent comprehensive survey of this topic was presented by Rohsenow and Hartnett in 1985. Since that time many solutions have been presented in literature. An extensive literature survey will discuss major solutions to date for direct transfer type heat exchangers (excluding shell and tube type). After identifying the ranges of parameters for which no solutions are available in literature, a finite difference model will be utilized to generate results for counterflow heat exchanger performance under transient conditions.

2. Theoretical Background

2.1 Idealizations.

In order to derive governing differential equations describing transient behavior of a heat exchanger the following idealizations are generally made (Shah, 1981).

1) The temperature of both fluids and the wall are functions of time and position.

2) Heat transfer between the exchanger and the surroundings is negligible. There are no thermal energy sources within the exchanger.

3) The mass flow rate of both fluids do not vary with time. Fluid passages are uniform in cross section giving a uniform fluid inventory in the heat exchanger.

4) The velocity and temperature of each fluid at the inlet are uniform over the flow cross section and are constant with time except for the imposed step change.

5) The convective heat transfer coefficient on each side, and the thermal properties of both fluids and the wall are constant, independent of temperature, time and position.

6) Longitudinal heat conduction within the fluids and wall as well as the transverse conduction within the fluid is neglected.

7) The heat transfer surface area on each fluid side is uniformly distributed in the heat exchanger.

8) Either the fouling resistances are negligible or they are lumped with thermal resistances of the wall.

9) The thermal capacitance of the heat exchanger enclosure is considered negligible relative to that of the heat transfer surface.

2.2 Governing differential equations.

In order to derive the governing differential equations, consider a counterflow heat exchanger and an incremental control volume shown in Figure 2.1.

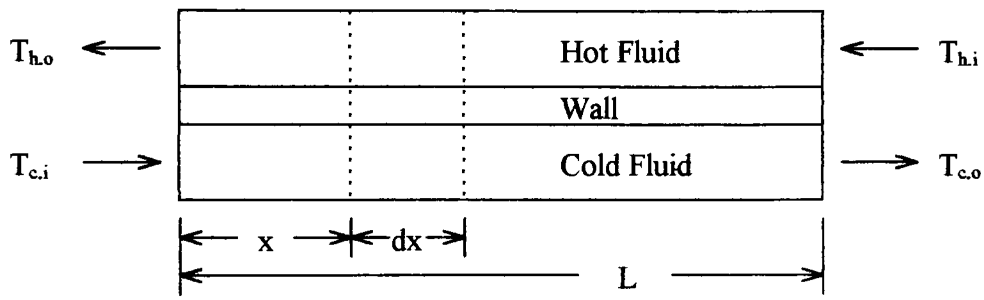


Figure 2.1 Counterflow heat exchanger and incremental control volume

Before continuing, some definitions are in order. Heat capacitance, $\overline{C}$, is defined as the product of mass and specific heat:

$$\text{Heat capacitance} = \overline{C} = Mc_p \tag{2.1}$$

Heat capacity rate, C, is the product of mass flow rate and specific heat:

$$\text{Heat capacity rate} = C = Wc_p \tag{2.2}$$

Applying an energy balance to incremental control volumes around the hot fluid, cold fluid, and wall yields the following differential equations after simplification (see Appendix A for further details):

$$\overline{C}_h \frac{\partial T_h}{\partial t} + C_h L \frac{\partial T_h}{\partial x} + (\eta_0 hA)_h (T_h - T_w) = 0 \tag{2.3}$$

$$\overline{C}_c \frac{\partial T_c}{\partial t} - C_c L \frac{\partial T_c}{\partial x} - (\eta_0 hA)_c (T_w - T_c) = 0 \tag{2.4}$$

$$\overline{C}_w \frac{\partial T_w}{\partial t} - (\eta_0 hA)_h (T_h - T_w) + (\eta_0 hA)_c (T_w - T_c) = 0 \tag{2.5}$$

Refer to the nomenclature for definition of various terms. In order to completely define the problem, initial and boundary conditions are required. The initial conditions are obtained from the steady state temperature distribution prior to the transient input. The boundary conditions are the input temperature change of the stepped fluid to its new value and the constant inlet temperature of the unstepped fluid.

Based on the above differential equations, the dependent fluid and wall temperatures are functions of the following variables and parameters:

$$T_h, T_c, T_w = f(x, T_{h,i}, T_{c,i}, C_h, C_c, (\eta_o hA)_h, (\eta_o hA)_c, \text{flow arrang.}, t, \overline{C_w}, \overline{C_h}, \overline{C_c}) \tag{2.6}$$

Eleven independent variables and parameters exist for the dependent fluid and wall temperatures for a given flow arrangement. Because the differential equations and boundary conditions are available, it is possible to formulate a set of dimensionless parameters by a purely analytical approach. These dimensionless parameters are not unique; the choice of the form of each dimensionless group is based on the usefulness to the designer. Cima and London (1958) define the following dimensionless parameters which are in widespread use today:

$$T^* = \frac{T(t) - T(0)}{T(\infty) - T(0)} \tag{2.7}$$

$$X^* = x/L = \text{dimensionless flow length variable} \tag{2.8}$$

$$NTU = UA/C_{min} = \text{number of transfer units} \tag{2.9}$$

$$C^* = C_{min}/C_{max} = \text{capacity rate ratio} \tag{2.10}$$

$$R^* = \frac{(\eta_o hA) \text{ on the } C_{max} \text{ side}}{(\eta_o hA) \text{ on the } C_{mm} \text{ side}} = \text{thermal resistance ratio} \tag{2.11}$$

$$t^* = t/t_{d,min} = \text{dimensionless time variable} \tag{2.12}$$

$$\overline{C_w}^* = \overline{C_w} / \overline{C_{min}} = \text{wall capacitance ratio} \tag{2.13}$$

$$t_d^* = \frac{t_{d,min}}{t_{d,max}} = \frac{t_d \text{ on the } C_{min} \text{ side}}{t_d \text{ on the } C_{max} \text{ side}} = \text{dwell time ratio} \tag{2.14}$$

We now have:

$$T_h^*, T_c^* = f(X^*, NTU, C^*, t^*, R^*, \overline{C_w}^*, t_d^*) \tag{2.15}$$

The variable X* representating the location within a heat exchanger can be eliminated because the temperature histories of primary interest in process applications are the outlet temperatures of each fluid which occur at either $X^* = 0$ or $X^* = 1$.

Some authors have proposed an alternate designation for the dependent dimensionless variables (We prefer to call them transient temperature effectiveness for sides 1 and 2):

$$\varepsilon_{f,1}{}^* = \frac{T_1(t) - T_1(0)}{T_1(\infty) - T_1(0)} \tag{2.16}$$

$$\varepsilon_{f,2}{}^* = \frac{T_2(t) - T_2(0)}{T_2(\infty) - T_2(0)} \tag{2.17}$$

Where the subscript 1 refers to the fluid which had a step input imposed on it and the subscript 2 refers to the unstepped fluid. All temperatures in equations (2.16) and (2.17) refer to outlet temperatures. With this nomenclature, the functional dependence can be stated as:

$$\varepsilon_{f,1}{}^*, \varepsilon_{f,2}{}^* = f(NTU, C^*, t^*, R^*, \overline{C_w}{}^*, t_d{}^*) \tag{2.18}$$

The reader can now appreciate the complexities involved in a transient heat exchanger problem since it requires a solution of three simultaneous partial differential equations for temperature as a function of time and position. The solution depends on six independent groups as well as exchanger flow arrangement. No general solution to this problem exists. Available solutions are usually restricted to certain values or ranges of independent variables.

Objectives of the present work are to present a thorough review of major solutions available in literature. Emphasis will be placed on the application of these solutions to practical engineering situations. A commercially available thermal network solver software package (Thermonet) will be utilized to model the transient response of heat exchangers. Thermonet will be utilized to generate solutions over a wide range of parameters covering cases for which no solution currently exists.

3. Literature Review

3.1 Introduction

Many solutions to the transient heat exchanger problem exist in literature. Typically, an investigator will utilize one or more of the following methods to obtain a solution.

- Direct analytical solution of the governing differential equations: Several idealizations and restrictions are incorporated to develop a closed form solution.
- Finite difference schemes: The governing differential equations can be modeled with computer based finite difference schemes. This eliminates the need for restrictions on independent variables. The data from this procedure is analyzed in an effort to develop some approximate, empirically based solutions.
- Electromechanical analog tests: The transient response of a heat exchanger is modeled using an equivalent electrical circuit consisting of capacitors to represent wall capacitance, and electric resistors to represent fluid flow and heat transfer resistances. Cima and London (1958) first utilized this method to develop some approximate solutions. A number of modifications to these solutions have been suggested in recent literature.

As was previously mentioned, the complexity of the problem generally prohibits the development of a general solution valid for all values of independent parameters. Most solutions are restricted to specific values or ranges of values of the independent variables. Most major solutions found in literature will be discussed in this paper. Limiting idealizations and restrictions will be discussed for each solution. In order to simplify presentation, solutions will be categorized as follows:

1) Solutions valid for all flow arrangements with $C^* = 0$ (see section 3.2 for explanation of $C^* = 0$)

2) Solutions valid for parallel flow arrangements.

3) Solutions valid for counter flow arrangements.

4) Solutions valid for crossflow arrangements.

3.2 Solutions for $C^* = 0$

In condensers and evaporators the heat capacity rate of one fluid is infinite (except for the pressure drop effect on saturation temperature), which means C^* is very small and can be approximated as zero. For these types of heat exchangers, the temperature of the C_{max} fluid can be approximated as constant throughout the exchanger. Solutions for these cases are valid for heat exchangers with any arrangements. Solutions of this type are further broken down into two categories:

- A step input change in the C_{min} fluid.
- A step input change in the C_{max} fluid.

Table 3.1 summarizes solutions found in literature which are valid for all flow arrangements with $C^* = 0$.

Table 3.1 Solutions for $C^* = 0$

Restrictions	Solution Method	Reference
Step change in C_{max} fluid		
$0 \le t^* \le 1$ $\overline{C_w}^* \le 1$	Analytical	Rizika (1956)
$R^* = \infty$ or $\overline{C_w}^* = 0$	Analytical	London et al.(1959)
$R^* = 0$ or $\overline{C_w}^* = 0$	Analytical	London et al.(1959)
$R^*=1$ and $\overline{C_w}^* > 5$	Electromechanical analog	London et al.(1959)
$R^*=1$ and $NTU=3$	Electromechanical analog	London et al.(1959)
$\overline{C_w}^*=20$ and $NTU=3$	Electromechanical analog	London et al.(1959)
$\overline{C_w}^* > 1$ $R^* > 1$ and $t^* > 1$	Finite Difference	Myers et al. (1970)
$\overline{C_w}^* > 100$	Analytical	Myers et al. (1970)
Step change in C_{min} fluid		
$R^* = 1$ and $\overline{C_w}^* > 20$	Electromechanical analog	London et al. (1959)
$R^* = 1$ and $NTU = 1$	Electromechanical analog	London et al. (1959)
$NTU = 1$ and $\overline{C_w}^* \ge 20$	Electromechanical analog	London et al. (1959)
$t^* > 1$	Analytical	Myers et al. (1967)

3.1.1 Step change in inlet temperature of C_{max} fluid.

In this case, sudden changes in the inlet temperature of the condensing or evaporating fluid occur due to sudden changes in the system pressure. The graphical representation of this scenario can be seen in Figure 3.1.

One of the early investigators to present a solution to this problem was Rizika (1956). He obtained an exact solution for $0 \le t^* \le 1$ and all values of $\overline{C_w}^*$ and R^*.

$$\varepsilon_{f,2}^* = \frac{1 - e^{-X}\left[Y\sinh(X/Y) + \cosh(X/Y)\right]}{1 - \exp(-NTU)} \tag{3.1}$$

$$X = \frac{NTU(1 + R^*)(1 + R^* + \overline{C}_w^*)t^*}{2R^*\overline{C}_w^*} \tag{3.2}$$

$$Y = \left[1 - \frac{4R^*\overline{C}_w^*}{(1 + R^* + \overline{C}_w^*)^2}\right]^{-1/2} \tag{3.3}$$

Shah (1981) in a comprehensive survey points out that in the above solution, a t^* between 0 and 1 is relevant only for cases with $\overline{C_w}^* \le 1$. London et al. (1959) present analytical solutions for the two limiting cases, $R^* = \infty$ and $R^* = 0$ as well as results for specific values of NTU, $\overline{C_w}^*$, and R^* based on electromechanical analog tests.

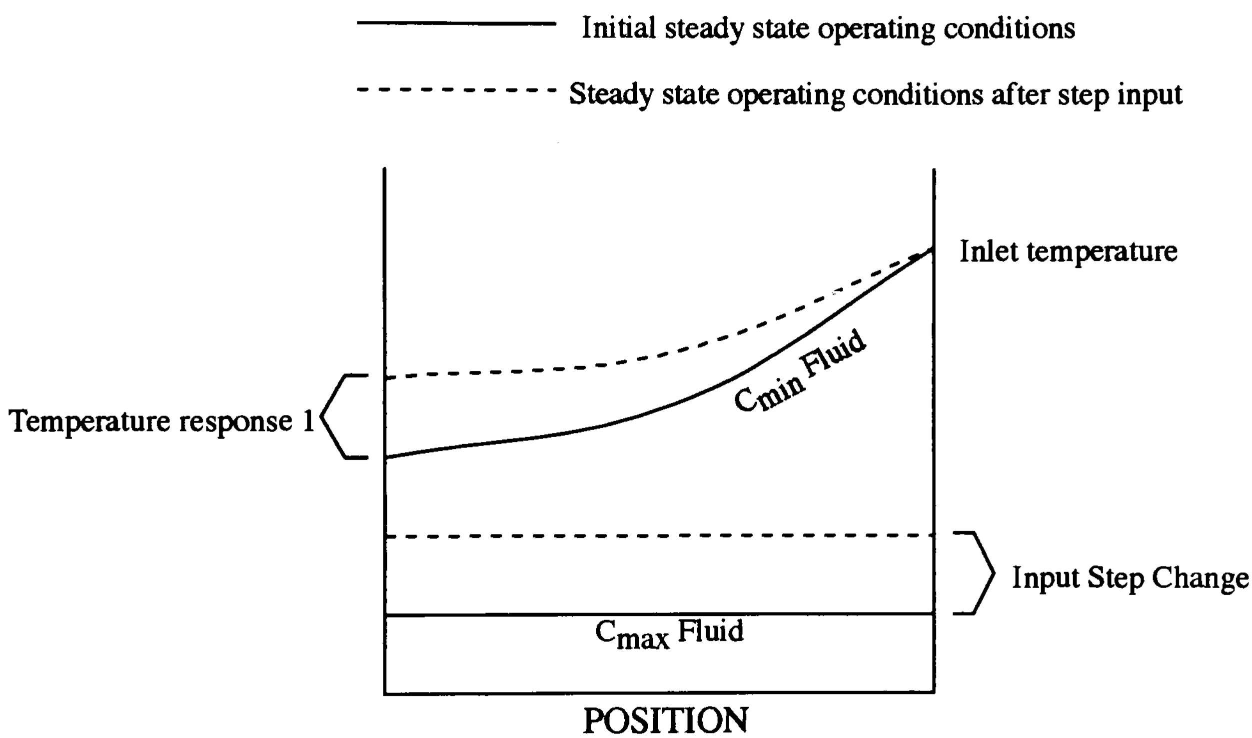


Figure 3.1 Typical temperature response for the C_{min} fluid for a step change in the inlet temperature of the C_{max} fluid.

Myers et al. (1970) obtained finite difference solutions for intermediate and large values of $\overline{C_w}^*$. Myers used data obtained from the finite difference solution to extrapolate the exact solution given by Rizika (1956). The results are as follows:

$$\varepsilon_{f,2}^* = 1 - A\exp\left[-B(t^* - 1)/\overline{C}_w^*\right] \tag{3.4}$$

$$A = 1 - \frac{1 - e^{-Z}\left[Y\sinh(Z/Y) + \cosh(Z/Y)\right]}{1 - \exp(-NTU)} \tag{3.5}$$

$$B = \frac{1}{A}\left[\frac{2NTU(1 + R^*)\overline{C}_w^*}{\left(1 + R^* + \overline{C}_w^*\right)}\right]\left[\frac{Ye^{-Z}\sinh(Z/Y)}{1 - \exp(-NTU)}\right] \tag{3.6}$$

$$Z = \frac{NTU(1 + R^*)\left(1 + R^* + \overline{C}_w^*\right)}{2R^*\overline{C}_w^*} \tag{3.7}$$

This solution shows excellent agreement with the finite difference solutions of Myers et al. (1970) within the following ranges:

$$t^* > 1$$

$$R^* \geq 1$$

The range for which the above solution by Myers is valid has been discussed in several comprehensive surveys on the subject of transient heat exchanger behavior. Kays and London (1964) report this range to be $\overline{C_w}^* > 100$. Shah (1981) states that the applicable range is $1 < \overline{C_w}^* \leq 2000$. Myers (1970) makes the statement that his solution is valid for "intermediate" values of $\overline{C_w}^*$. A detailed study was undertaken in this thesis to clarify this apparent discrepancy. It has been shown conclusively in Appendix B that the above solution by Myers is valid for all values of $\overline{C_w}$ * greater than 1. It should be noted that Myers (1970) states as one of his assumptions that the temperature of the infinite capacitance rate fluid is initially at zero when it is suddenly stepped to a value T at time t = 0. This solution by Myers eliminates many of the restrictions utilized by the previous solutions of London et al (1959).

Myers also proposes an analytical solution for $\overline{C_w}^* > 100$ making extensive use of Bessels functions.

3.1.2 Step change in the inlet temperature of the C_{min} fluid.

A graphical representation of this scenario case is shown in Figure 3.2. London et al. (1959) have developed approximate solutions for this case for specific values of NTU, $\overline{C_w}^*$ and R* These solutions are approximate and were obtained from electromechanical analog test results. The paper by Myers et al. (1967) presents an analytical solution to the more general problem of a crossflow heat exchanger where C* is not constrained to be zero. Myers applies the constraint of C* = 0 to his more general solution and determines that solution is a function of two independent variables, namely;

$$NTU/R^*$$

$$(1+R^*)^2(t^*-1)/\overline{C_w}^*$$

The solution can be obtained directly from the analytical expression given by Myers but it again involves integration of Bessels functions and is inconvenient to use. Fortunately Myers has supplied a convenient graphical solution which is easy to utilize. As seen from

Figure 3.3, the exact solution for a step change in the noncondensing or evaporating fluid can be represented on one graph.

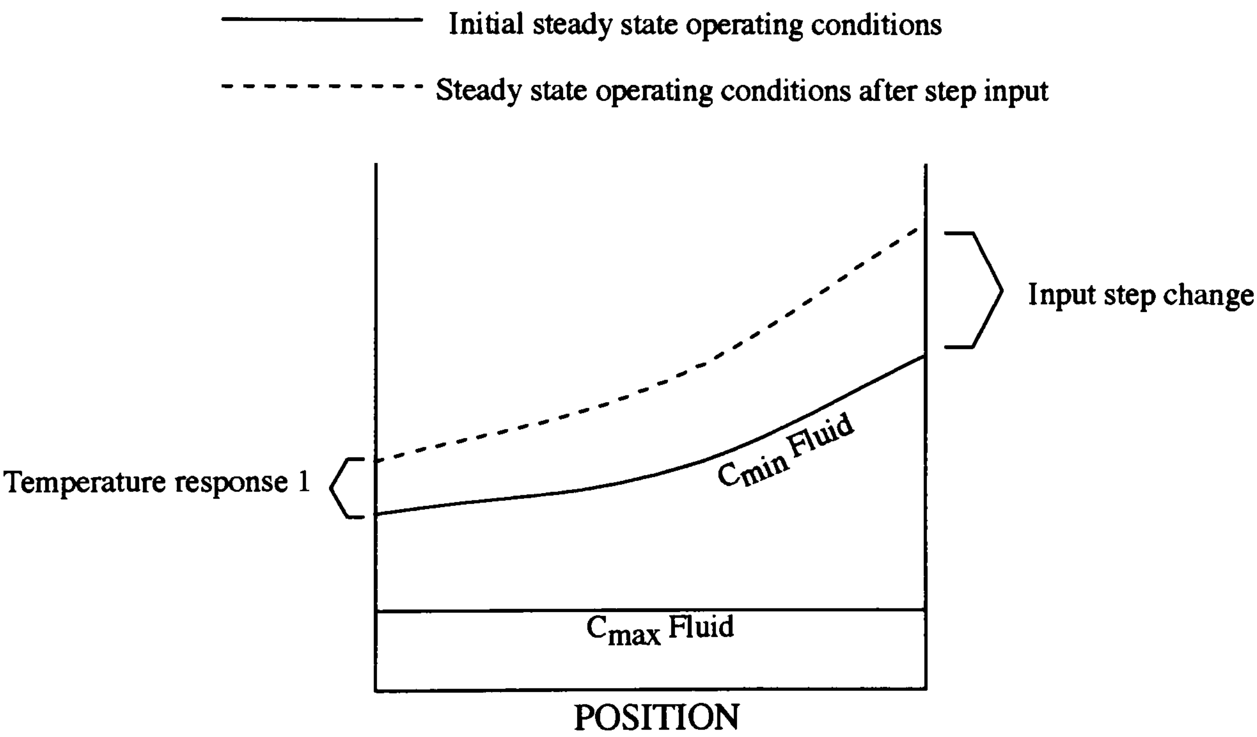


Figure 3.2 Typical temperature response of the C_{min} fluid for a step change in the temperature of the C_{min} fluid.

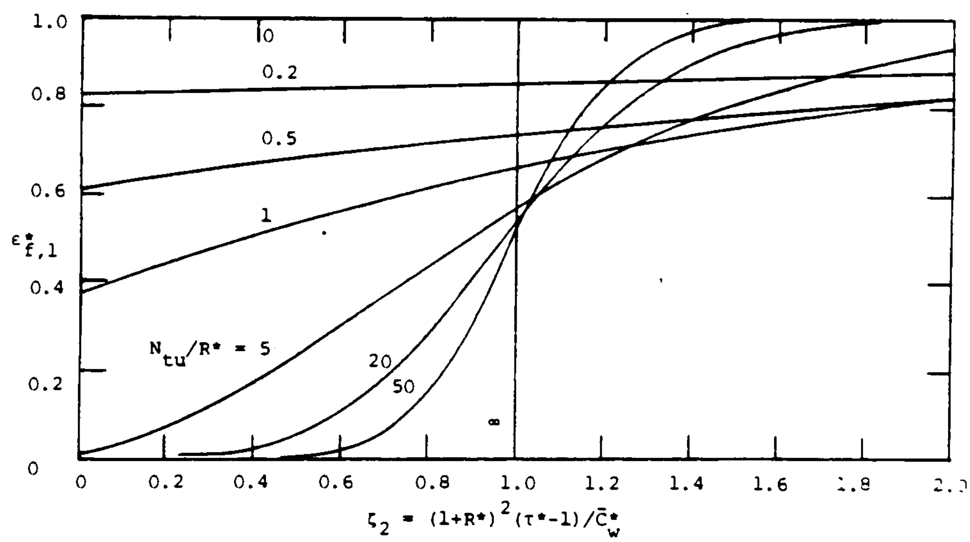


Figure 3.3. Outlet temperature response for a step change in the inlet temperature of the C_{min} fluid, Myers (1967)

3.3 Solutions for Parallel Flow Configurations

Table 3.2 summarizes solutions found in literature for parallel flow configurations.

Table 3.2 Solutions for parallel flow configurations.

Restrictions	Solution Method	Reference
Dwell time of fluids are equal or both fluids are gases	Analytical	Romie (1985)
Thermal capacitance of the core is assumed to be negligible compared to the thermal capacitance of the stepped fluid.	Analytical	Li (1986)
Both fluids must be gases.	Analytical	Gvozdenac (1987)

Romie (1985) presented a solution for a parallel flow heat exchanger. The solution to the transient problem is presented as a function of six new dimensionless groups defined as follows:

$$X = \frac{x}{L} \tag{3.8}$$

$$\theta = \left(t - \frac{x}{v_1} \right) \left(\frac{C_{\min}}{\overline{C}_{wall}} \right) \tag{3.9}$$

$$E = \frac{C_2}{C_1} \tag{3.10}$$

$$R = \frac{(hA)_2}{(hA)_1} \tag{3.11}$$

$$\frac{1}{NTU} = C_{\min} \left[\frac{1}{(hA)_1} + \frac{1}{(hA)_2} \right] \tag{3.12}$$

$$V = \frac{C_{\min}}{\overline{C}_{wall}} \left[\frac{L}{v_2} - \frac{L}{v_1} \right] \tag{3.13}$$

Where v is the velocity and the subscript 1 refers to the stepped fluid and the subscript 2 refers to the unstepped fluid. Romie introduced the constraint that the dimensionless parameter V as defined by equation 3.13 must be zero. As can be seen from equation 3.13, this will be valid if the velocities of the two fluid streams are equal. Alternatively, if both fluids are gases then the absolute value of V will be very small and can be equated to zero whether or not the velocities are equal. With this constraint, the temperature response is a function of four independent variables, namely θ, E, R, and NTU (see equations 3.9-13). Romie utilizes Laplace transforms to obtain an analytical solution. The solution is very complex and involves integration of Bessel functions. Romie presents several graphs of solutions for specific values of the dimensionless groups.

Solutions for parallel flow heat exchangers are also proposed by Li (1986). Li introduces the idealization that the thermal capacitance of the core is negligible compared to the thermal capacitance of the stepped fluid. This idealization will be valid when the stepped fluid is a liquid. In a steel heat exchanger, the specific heat of water is about ten times the specific heat of steel, hence the thermal capacitance of the core may be negligible in many cases. In general, Li's solution is valid for liquid to liquid heat exchangers, or liquid to gas heat exchangers where the liquid is the stepped fluid. The fluid velocities need not be equal. Analytical solutions are obtained for the case where the velocities of the two fluids are equal and for the case where the velocities are not equal. The solution for the case of unequal velocities is again very complex involving integration of Bessel functions. However, the solution for the case of equal velocities is relatively straightforward.

Gvozdenac (1987) also presents a solution for parallel flow heat exchangers. His solutions are restricted to the cases where the thermal capacities of the two fluids are negligible relative to the thermal capacity of the heat exchanger core. This restriction will be valid if fluids are gases. Solutions are derived by using the method of successive approximations and the Laplace transform method. A significant feature of this work is that it is valid for any type of input change in temperature (step function, sinusoidal, exponential, etc.).

3.4 Solutions for Counterflow Configurations

Table 3.3 summarizes solutions found in literature which are valid for counterflow configurations.

Table 3.3 Solutions valid for counterflow configurations.

Restrictions	Solution Type	Reference
$C^* = 1$ $R^* = 1$ $10 < \overline{C_w}^* < 40$ $.6 \le NTU \le 8$	Electromechanical analog	Cima and London (1958)
$C^* = 1$ $\overline{C_w}^* > 100$ $1.5 \le NTU \le 8$ $\varepsilon_{f,2}^*$ only	Electromechanical analog	London et al. (1964)
$C^* = 1$ $\overline{C_w}^* > 100$ $1.5 \le NTU \le 6$ $.25 \le R^* \le 4$ $\varepsilon_{f,1}^*$ only	Electromechanical analog	London et al. (1964)
See below	Finite difference	Romie (1984)
Both fluids gases	Analytical	Gvozdenac (1987)

Cima and London (1958) investigated the transient response of counterflow heat exchangers used for a specific application - namely a gas turbine regenerator. Heat exchangers used for this purpose can be closely approximated as having $C^* = 1$ and $R^* = 1$. These restrictions reduce the number of independent variables to four:

$$\varepsilon_{f,1}^*, \varepsilon_{f,2}^* = f(NTU, \overline{C_w}^*, t^*, t_d^*) \tag{3.14}$$

They further simplify the problem by only solving for the temperature response of the unstepped fluid, $\varepsilon_{f,2}^*$. Cima and London discovered that $\varepsilon_{f,2}^*$ was virtually insensitive to variations in t_d^* for $\overline{C_w}^* > 10$. It was also noted that an empirical correlation of $\varepsilon_{f,2}^*$

versus $t^*/(1.5+\overline{C_w}^*)$ accurately predicted transient response for $0 < \overline{C_w}^* < 50$. They were thus able to reduce the problem to two independent parameters:

$$\varepsilon_{f,2}^* = f(NTU, t^*/(1.5+\overline{C_w}^*)) \quad (3.15)$$

London et al. (1964) again utilize electromechanical analog results to provide more solutions to this problem. They provide solutions for the stepped fluid temperature response ($\varepsilon_{f,1}^*$) in addition to the unstepped fluid response ($\varepsilon_{f,2}^*$). The experiments conducted by London et al. demonstrate convincingly that R^* in the range $1/4 \leq R^* \leq 4$ have no measurable influence on $\varepsilon_{f,2}^*$. All the above results can be depicted in one graph as seen in Figure 3.4.

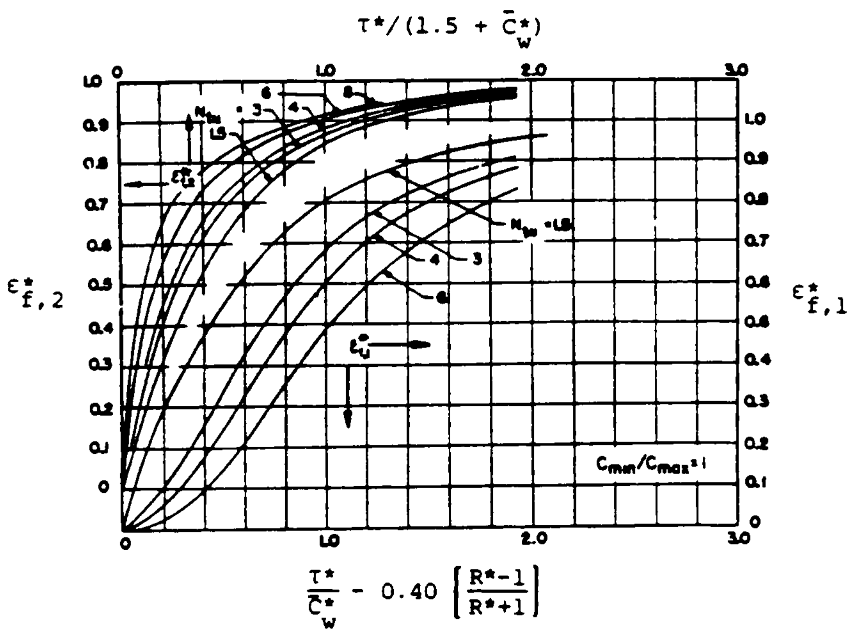


Figure 3.4 Solutions for counterflow configurations, London et al. (1964).

Romie (1984) utilizes a finite difference method to propose a simple empirical relation using slightly different dimensionless groups than he used for parallel flow:

$$X = \frac{x}{L} \tag{3.16}$$

$$\theta_1 = \left[t - \frac{L}{v_1}\right]\left[\frac{C_{\min}}{\overline{C}_{wall}}\right] \tag{3.17}$$

$$\theta_2 = (t)\left[\frac{C_{\min}}{\overline{C}_{wall}}\right] \tag{3.18}$$

$$E = \frac{C_2}{C_1} \tag{3.19}$$

$$R = \frac{(hA)_2}{(hA)_1} \tag{3.20}$$

$$\frac{1}{NTU} = C_{\min}\left[\frac{1}{(hA)_1} + \frac{1}{(hA)_2}\right] \tag{3.21}$$

$$V = \frac{C_{\min}}{\overline{C}_{wall}}\left[\frac{L}{v_2} + \frac{L}{v_1}\right] \tag{3.22}$$

Where v is velocity and the subscript 1 refers to the stepped fluid and the subscript 2 refers to the unstepped fluid. The dimensionless parameters as defined by Romie are related to the dimensionless parameters previously discussed as follows:

$$R = \frac{1}{R*} \tag{3.23}$$

$$V = \frac{1}{\bar{C}_w^*}\left[1 + \frac{1}{t_d^*}\right] \tag{3.24}$$

$$\theta_2 = \frac{t^*}{\bar{C}_w^*} \tag{3.25}$$

If step change is on C_{min} side:

$$\theta_1 = \frac{t^* - 1}{\bar{C}_w^*} \tag{3.26}$$

$$E = \frac{1}{C^*} \tag{3.27}$$

If step change is on C_{max} side:

$$\theta_1 = \frac{t^* t_d^* - 1}{\bar{C}_w^* t_d^*} \tag{3.28}$$

$$E = C^* \tag{3.29}$$

Romie presents a solution in compact empirical form:

$$\frac{T_1(t)}{T_1(\infty)} = 1 - Ae^{-\frac{a\,\theta_1}{1+V}} - Be^{-\frac{b\,\theta_1}{1+V}} \tag{3.30}$$

$$\frac{T_2(t)}{T_2(\infty)} = 1 - Ce^{-\frac{c\,\theta_2}{1+V}} - De^{-\frac{d\,\theta_2}{1+V}} \tag{3.31}$$

Table 3.4 Constants utilizied in solution by Romie (1984)

B	R	V	A / C	a / c	B / D	b / d	A / C	a / c	B / D	b / d	A / C	a / c	B / D	b / d	A / C	a / c	B / D	b / d
			Ntu = 1				Ntu = 2				Ntu = 4				Ntu = 8			
0.8	0.5	0	1.154	1.96	-0.311	3.82	1.760	2.16	-0.780	4.40	2.122	1.82	-1.122	3.88	2.282	1.28	-1.282	2.90
			0.937	2.01	0.063	4.41	0.734	2.20	0.266	6.17	0.479	1.86	0.521	7.90	0.276	1.36	0.724	9.24
0.8	1.0	0	0.686	1.83	-0.034	4.31	1.119	2.02	-0.213	5.08	1.567	1.74	-0.572	4.59	1.878	1.23	-0.878	3.29
			0.913	1.87	0.087	4.24	0.712	2.07	0.288	5.74	0.472	1.77	0.528	7.55	0.278	1.31	0.722	9.02
0.8	2.0	0	0.476	1.93	0.005	3.20	0.827	2.07	-0.038	5.42	1.220	1.72	-0.246	5.56	1.584	1.20	-0.585	3.76
			0.977	1.94	0.023	5.02	0.749	2.08	0.251	5.81	0.487	1.75	0.513	7.48	0.283	1.28	0.717	8.85
1.0	0.5	0	1.346	1.76	-0.446	3.50	2.003	1.86	-1.010	3.82	2.245	1.48	-1.245	3.28	2.245	0.97	-1.245	2.26
			0.926	1.82	0.074	4.05	0.701	1.88	0.299	5.61	0.441	1.49	0.559	6.99	0.255	0.99	0.745	8.15
1.0	1.0	0	0.807	1.69	-0.077	3.78	1.248	1.80	-0.399	4.39	1.676	1.45	-0.678	3.74	1.885	0.95	-0.885	2.49
			0.889	1.73	0.111	4.00	0.680	1.83	0.320	5.52	0.462	1.51	0.538	7.73	0.270	1.02	0.730	8.44
1.0	2.0	0	0.554	1.81	-0.001	7.84	0.922	1.86	-0.071	5.37	1.305	1.47	-0.317	4.85	1.641	0.96	-0.641	2.88
			0.926	1.82	0.074	4.05	0.701	1.88	0.299	5.61	0.441	1.49	0.559	6.99	0.255	0.99	0.745	8.15
1.25	0.5	0	1.327	1.85	-0.432	3.76	2.108	2.07	-1.117	4.05	2.450	1.75	-1.450	3.59	2.550	1.20	-1.550	2.45
			0.977	1.94	0.023	5.02	0.749	2.08	0.251	5.81	0.487	1.75	0.513	7.48	0.283	1.28	0.717	8.85
1.25	1.0	0	0.788	1.81	-0.074	4.20	1.315	2.04	-0.379	4.51	1.800	1.76	-0.802	4.24	2.115	1.25	-1.115	2.92
			0.913	1.87	0.087	4.24	0.712	2.07	0.288	5.74	0.472	1.77	0.528	7.55	0.278	1.31	0.722	9.02
1.25	2.0	0	0.533	2.01	-0.003	3.98	0.900	2.18	-0.072	5.79	1.316	1.84	-0.333	5.51	1.730	1.30	-0.730	3.40
			0.937	2.01	0.063	4.41	0.734	2.20	0.266	6.17	0.479	1.86	0.521	7.90	0.276	1.36	0.724	9.24
1	1	1	0.701	2.61	0.028	13.75	0.989	2.28	-0.440	12.03	1.293	1.61	-0.295	6.57	1.562	0.98	-0.562	3.39
			1.356	2.61	-0.356	8.42	0.872	2.28	0.128	5.02	0.478	1.61	0.513	7.59	0.273	1.03	0.727	8.59
1	1	2	0.599	3.03	0.130	13.19	0.868	2.45	0.077	12.14	1.157	1.65	-0.158	8.66	1.443	0.99	-0.443	3.66
			1.739	3.06	-0.739	6.24	1.000	2.49	0	--	0.509	1.66	0.491	7.98	0.261	1.01	0.739	8.33
1	1	4	0.478	3.30	0.251	17.80	0.768	2.58	0.177	21.40	1.058	1.68	-0.059	9.10	1.352	0.99	-0.352	4.10
			2.400	3.50	-1.400	5.63	1.000	2.52	0	--	0.518	1.68	0.482	8.12	0.262	1.01	0.738	8.32
1	1	8	0.422	3.53	0.307	31.79	0.698	2.65	0.256	36.56	0.998	1.70	0	--	1.313	1.00	-0.313	4.13
			2.750	3.74	-1.750	5.62	1.144	2.66	-0.144	2.87	0.522	1.69	0.478	8.19	0.262	1.01	0.738	8.29
0.8	0.5	4	0.469	3.53	0.374	13.34	0.760	2.86	0.220	14.29	1.000	1.91	0	--	1.364	1.21	-0.364	4.91
			3.790	4.09	-2.790	5.55	1.323	2.92	-0.323	4.65	0.581	1.92	0.419	8.67	0.280	1.23	0.720	8.55
0.8	1.0	4	0.397	3.40	0.255	18.52	0.702	2.83	0.204	34.18	0.979	1.89	0.015	8.48	1.311	1.20	-0.311	4.99
			3.000	3.80	-2.000	5.62	1.223	2.83	-0.223	5.31	0.578	1.90	0.422	8.53	0.324	1.32	0.676	9.50
0.8	2.0	4	0.331	3.27	0.149	27.93	0.635	2.80	0.154	42.77	0.936	1.89	0.038	99.00	1.268	1.19	-0.268	5.03
			3.620	3.95	-2.620	5.40	1.334	2.89	-0.334	4.55	0.574	1.88	0.426	7.90	0.282	1.22	0.718	8.50
1.0	0.5	4	0.554	3.35	0.346	10.62	0.841	2.59	0.151	7.50	1.122	1.68	-0.122	11.85	1.405	1.00	-0.405	3.96
			2.800	3.67	-1.800	5.38	1.074	2.58	-0.074	3.67	0.520	1.68	0.480	8.16	0.262	1.01	0.738	8.30
1.0	1.0	4	0.478	3.30	0.251	17.80	0.768	2.58	0.177	21.40	1.058	1.68	-0.059	9.10	1.352	0.99	-0.352	4.10
			2.400	3.50	-1.400	5.63	1.000	2.52	0	--	0.518	1.68	0.482	8.12	0.262	1.01	0.738	8.32
1.0	2.0	4	0.400	3.24	0.153	33.89	0.698	2.58	0.152	40.36	0.999	1.68	-0.012	4.87	1.307	0.99	-0.307	4.23
			2.800	3.67	-1.800	5.38	1.074	2.58	-0.074	3.67	0.520	1.68	0.480	8.16	0.262	1.01	0.738	8.30
1.25	0.5	4	0.518	3.54	0.377	10.79	0.795	2.82	0.196	8.62	1.083	1.90	-0.083	38.40	1.396	1.20	-0.396	4.75
			3.620	3.95	-2.620	5.40	1.334	2.89	-0.334	4.55	0.574	1.88	0.426	7.90	0.282	1.22	0.718	8.50
1.25	1.0	4	0.417	3.41	0.298	16.13	0.720	2.83	0.216	23.48	1.013	1.91	-0.015	5.51	1.367	1.22	-0.367	4.53
			3.000	3.80	-2.000	5.62	1.223	2.83	-0.223	5.31	0.578	1.90	0.422	8.53	0.324	1.32	0.676	9.50
1.25	2.0	4	0.343	3.34	0.187	25.12	0.645	2.85	0.183	36.40	0.982	1.95	0	--	1.295	1.22	-0.295	4.83
			3.790	4.09	-2.790	5.55	1.323	2.92	-0.323	4.65	0.581	1.92	0.419	8.67	0.280	1.23	0.720	8.55

T(t) represents the outlet temperature at time t and T(∞) represents the final steady state outlet temperature after the step input has been imposed. Romie presents the constants A,a,B,b,C,c,D,d in tabular format as can be seen in Table 3.4.

It should be noted that this solution by Romie was derived by assuming the initial inlet temperatures of both fluid streams are 0°C and that the stepped inlet temperature is 1°C. These idealizations are not as restrictive as they appear. His solution can be utilized for any values of inlet temperatures if the left hand side of equations 3.30 and 3.31 are replaced with the respective dimensionless temperature expressions:

$$\frac{T_1(t) - T_1(0)}{T_1(\infty) - T_1(0)} = 1 - Ae^{-\frac{a\,\theta_1}{1+V}} - Be^{-\frac{b\,\theta_1}{1+V}} \tag{3.32}$$

$$\frac{T_2(t) - T_2(0)}{T_2(\infty) - T_2(0)} = 1 - Ce^{-\frac{c\,\theta_2}{1+V}} - De^{-\frac{d\,\theta_2}{1+V}} \tag{3.33}$$

T(0) represents the steady state outlet temperature before the step input has been imposed. The values of T(0) and T(∞) can be calculated using steady state analytical solution methods such as the effectiveness-NTU method. As can be seen from Romies tabular data, most of the solutions are for either V = 0 OR V = 4. For realistic values of t_d* between .25 and 4.0 it can be seen from equation 3.24 that in order for V to be equal to zero, C_w* must be infinitely large. This could be closely approximated by some gas to gas heat exchangers. In order for V to be equal to 4.0 while still maintaining realistic values of t_d*, C_w* must be between .3 and 1.3. This leads to the conclusion that Romies solution is valid over a somewhat limited range of C_w* values.

Romies solution produces inaccurate results for the stepped fluid for times less than one dwell time. The cause of this can be seen in equation 3.17. The term L/v_1 in this equation represents the dwell time of the stepped fluid. For values of dwell time greater than t, the value of θ will be negative which will lead to unrealistic temperatures when substituted into equation 3.32.

Gvozdenac (1987) presents an analytical solution for counterflow heat exchangers with the only restriction that the thermal capacities of the two fluids are negligible relative to the thermal capacity of the heat exchanger core. This restriction will be met if both fluids are gases with heavy separating walls. The smallness of this capacity ratio physically represents the fact that the fluid dwell times are small compared to the duration of the transient. The solution proposed by Gvozdenac is valid for any type of input change in temperature (step function, sinusoidal, etc.) The practical application of the explicit analytical solutions is somewhat complicated. Gvozdenac suggests that a numerical integration scheme can be implemented to calculate solutions. See Gvozdenac (1987) for further details.

3.5 Solutions for Crossflow Configurations

Table 3.5 summarizes solutions found in literature which are valid for crossflow heat exchanger configurations.

Table 3.5 Solutions for crossflow configurations

Restrictions	Solution Method	Reference
	Finite Difference	Dusinberre (1959)
One fluid mixed, other unmixed mixed fluid stepped $\overline{C_w}^{*}$ large	Analytical	Myers et al. (1967)
Both fluids unmixed	Finite Difference	Yamashita et al. (1978)
Both fluids gases Both fluids unmixed	Analytical	Romie (1983)
Both fluids gases Both fluids unmixed	Analytical	Gvozdenac (1986)
Both fluids gases Both fluids unmixed	Analytical	Spiga,Spiga (1987)
Delta input only Both fluids unmixed	Analytical	Spiga, Spiga (1988)
Both fluids gases Both fluids unmixed	Analytical	Chen, Chen (1991)
Delta input only Both fluids unmixed	Analytical	Chen, Chen (1992)
Both fluids unmixed	Analytical	Spiga, Spiga (1992)

One of the first researchers to investigate the transient behavior of crossflow heat exchangers was Dusinberre (1959) who proposed a finite difference method to describe the transient behavior. Very straightforward finite difference equations are presented which can easily be incorporated into a spreadsheet or computer program. Gas to gas heat exchangers are the primary focus of the paper by Dusinberre, however, equations are presented in the appendix of his paper which can be used when one of the fluid streams is a liquid. Dusinberre only considers one specific case and does not verify his solutions with any other source.

Myers et al. (1967) analyzed the transient response of a crossflow heat exchanger with one fluid mixed and the other fluid unmixed. Solutions are presented for the case where the mixed fluid has a temperature change applied to it. The solution also requires that the value of $\overline{C_w}^*$ be "large". Myers et al. propose that $\overline{C_w}^*$ will be sufficiently large if it meets the following condition:

$$\frac{a(1+R^*)^2 C^*}{(a+R^*)\overline{C}_w^*} \leq 05 \tag{3.19}$$

$$a = 1 + \frac{(e^{-N} - 1)}{N} \tag{3.20}$$

$$N = \frac{t_{d,2}}{\left[\frac{1}{(hA)_1} + \frac{1}{(hA)_2}\right] C_2} \tag{3.21}$$

The analytical solution presented by Myers et al. is quite complex, involving integration of Bessel functions. In addition, compact graphical representation of their results is not feasible due to the large number of independent variables. The authors present a useful graph (Figure 3.5) which can be used to determine the time required to attain a 90% response for $t^* > 10$ and large $\overline{C_w}^*$. The variables used in Figure 3.5 are defined as follows:

$$A' = \frac{M(a+R^*)}{1+R^*} \tag{3.22}$$

$$B = \frac{M(1+R^*)}{\overline{C}_w^*(a+R^*)} \tag{3.23}$$

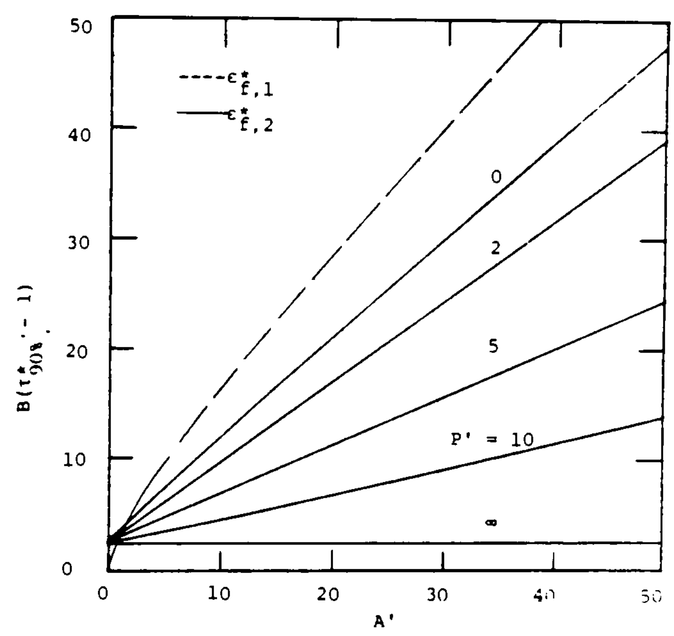


Figure 3.5 90% response times for a crossflow heat exchanger Myers et al. (1967)

$$P' = \frac{M(1-a)}{1+R^*} \tag{3.24}$$

$$M = \frac{t_{d,1}}{\frac{1}{(hA)_1}\overline{C}_1} \tag{3.25}$$

Yamashita et al. (1978) analyzed the transient response of a crossflow exchanger with both fluids mixed. A finite difference method was utilized to present graphical solutions for specific values of independent variables. They considered one of five groups (t_d^*, $1/\overline{C_w}^*$, R^*, NTU, C^*) as a variable while fixing the value of the others to be unity.

Romie (1983) proposes a solution utilizing the Laplace transform method. His results are valid for crossflow exchangers with neither gas mixed. The idealization is made that the thermal capacities of the masses of the two fluids contained in the exchanger are negligibly small relative to the thermal capacity of the exchanger core. As previously discussed, this will be true if both fluids are gases. The form of the solution is quite complex, requiring computer implementation. Romie does present some very useful graphical solutions which encompass the following ranges (see equations 3.19 - 3.21 for definition of these parameters):

$$1 \leq \text{NTU} \leq 8$$

$$.6 \leq E \leq 1.67$$

$$.5 \leq R \leq 2.0$$

Gvozdenac (1986) and Spiga and Spiga (1987) present a more general analytical solution which allow arbitrary initial and inlet conditions. The solutions are only valid for gases. In a later paper, Spiga and Spiga (1988) propose a solution where fluids can be gases or liquids. Their results are valid only for a deltalike (impulse) change in the inlet temperature of one of the fluids.

Due to the complexity of the numerical results presented by Spiga and Spiga (1987, 1988) computer implementation is required which itself can be a formidable task. Chen and Chen (1991, 1992) propose straightforward computer implementation techniques which increase computational efficiency.

Spiga and Spiga (1992) propose an exact analytical solution to the transient response due to a step change in inlet temperature of the hot fluid. Fluids are not constrained to be gases.

3.6 Recommendations from Available Solutions

There are many solutions available in literature to the transient heat exchanger problem. However, each solution is valid over a limited range of independent parameters. Some solutions derived from an analytical approach are valid for a wide range of independent parameters but the complex form of the solutions makes their application unlikely in a design situation. The following information will attempt to direct the reader toward a solution that not only is valid for a wide range of parameters but can also be conveniently utilized.

3.6.1 Solutions for $C^* = 0$.

For a step change in the C_{max} fluid, the approximate solution by Myers et al. (1970) is valid for a broad range of parameters. The solution has been verified by finite difference solutions by Myers et al. The form of the solution is very straightforward (see equations 3.4 through 3.7) and can be easily implemented in a design situation.

For a step change in the C_{min} fluid, the solution by Myers et al. (1967) is very useful due to its wide range of application. The explicit analytical solution is cumbersome, involving integration of Bessel functions. The graphical representation of Myers solution (Figure 3.3) can be conveniently utilized to obtain results.

3.6.2 Solutions for Parallel flow Configurations

Most available solutions to date are derived by analytical means and as a result the final solutions are very complex to utilize. However, Romie (1985) and Li (1986) present several graphical representations of their solutions for specific values of independent variables. If the values of the specific independent variables match those of a certain application, the graphs can be used to determine transient response.

3.6.3 Solutions for Counterflow Configurations

The electromechanical analog results of London et al. (1964) shown in Figure 3.4 can be utilized if $C^* = 1$ and other restrictions are met (see Table 3.3). The empirical solution by Romie (1984) covers a wider range of parameters while maintaining a simple empirical expression (equations 3.32, 3.33). As previously discussed, Romie's solution for the response of the stepped fluid is valid for times greater than one dwell time.

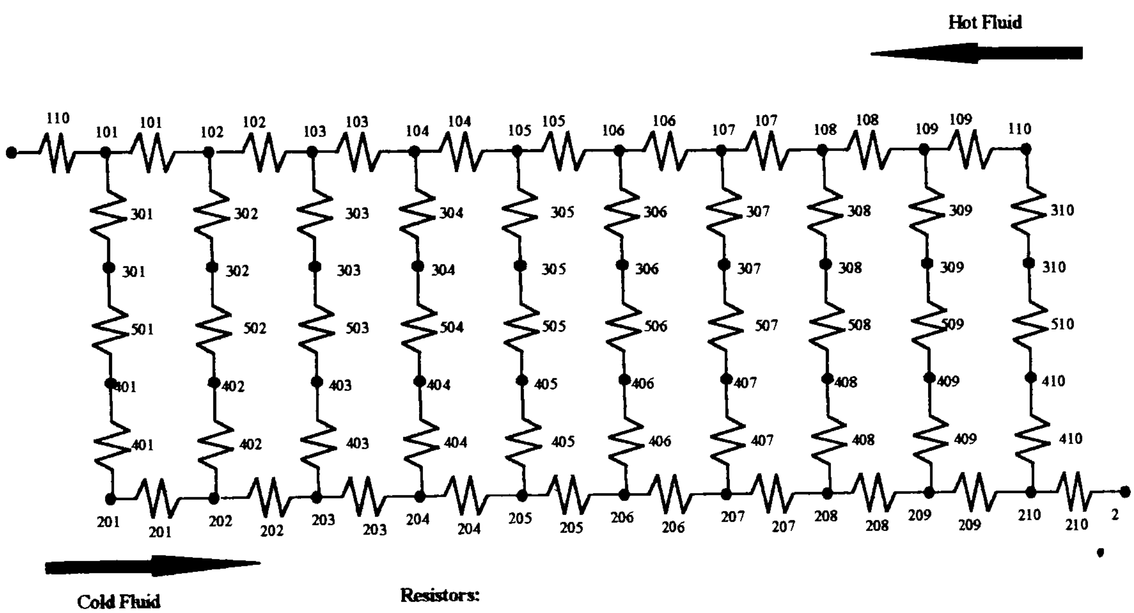
3.6.4 Solutions for Crossflow Configurations

As with parallel flow, crossflow heat exchangers have been analyzed by primarily analytical techniques, with the resulting solutions being cumbersome to implement. For both fluids being unmixed gases, Romie (1983) presents several graphical representations of his analytical solutions. These graphical solutions cover a fairly wide range of independent parameters.

The only solution available for the transient response to a step change when one or more of the fluids are liquids is by Spiga and Spiga (1992). Some graphical representations of the analytical solution are presented but are valid only for a very limited range of independent parameters.

4. Application of Thermal Network Solver

The transient response of a heat exchanger subjected to a step inlet temperature change of one of the fluids was modeled using "Thermonet" - a commercially available thermal network solver. Thermonet utilizes a finite difference algorithm to solve transient heat transfer problems. A heat exchanger can be modeled by discretizing it's length into a fixed number of segments. Fluid convection resistance, wall conduction resistance and fluid flow capacitance are modeled for a counterflow heat exchanger as shown in Figure 4.1.



Resistors:

101 - 110 Hot Fluid Flow Capacitance 1/mCp
201 - 210 Cold Fluid Flow Capacitance 1/mCp
301 - 310 Hot Fluid Convective Resistance 1/hA
401 - 410 Cold Fluid Convective Resistance 1/hA
501 - 510 Wall Resistance t/kA

Figure 4.1 Thermal network model of a counterflow heat exchanger

In order to determine the optimum number of segments that should be used in the thermal network, three models utilizing 5, 10, and 20 segments compared. For each case, the steady state performance calculated by Thermonet is compared to the exact steady state performance derived from effectiveness -NTU relations. As can be seen in Table 4.1, a 10 segment model shows very little deviation from theoretical (0.1% or less) for NTU $\leq$ 1.0. However, for NTU > 1.0 the 10 segment model shows deviations greater than 1%. For values of NTU > 1.0, the 20 segment model has better accuracy, with deviations generally less than 1%. Based on this information, a 10 segment model will be sufficient for NTU $\leq$ 1.0, while a 20 segment model should be utilized for 1.0 < NTU $\leq$ 5.0.

Once the optimum number of segments is determined, it is necessary to choose an appropriate time step in transient analysis. The optimum time step is one that achieves the desired accuracy without requiring excessive computation time. Since Thermonet is based on a finite difference scheme, one would expect that the smaller the time step, the more accurate the solution. This, however, is not the case as can be seen in Figures 4.2 - 4.4 of temperature vs. time step. The data in each of these figures was obtained by varying the chosen time step utilized by Thermonet, while keeping all other parameters constant. At very small time steps, erratic behavior is observed. This behavior is likely due to computational roundoff error propagation. The solution at these small time steps is most likely in error. As a result, very small time steps should be avoided. As a general rule of thumb, time steps equal to or larger than one half of the dwell time of the C_{min} fluid have been found to be reasonable. Decreasing the time step by a factor of 10 from its largest value results in a deviation of approximately 0.2 degrees C. A further reduction in time step by a factor of 10 lead to much larger deviations.

Table 4.1 Optimum number of segments

NTU	C*	Eff.	Inlet Temp (Degrees C)	Outlet Temperatures (Degrees C) Thermonet 5-Segment	10-Segment	20-Segment	E-NTU	Deviation from Theoretical (%) 5-Segment	10-Segment	20-Segment
0.50	0.50	0.362	100	71.098	71.036	71.021	71.019	0.1116	0.0243	0.0032
			20	34.451	34.482	34.490	34.491	0.1149	0.0250	0.0018
	1.00	0.333	100	73.449	73.36	73.338	73.333	0.1577	0.0364	0.0064
			20	46.551	46.64	46.662	46.667	0.2479	0.0571	0.0100
1.03	0.50	0.574	100	54.414	54.087	54.024	54.082	0.6138	0.0092	0.1073
			20	42.793	42.957	42.988	42.959	0.3864	0.0046	0.0676
	1.00	0.507	100	59.877	59.47	59.36	59.409	0.7880	0.1029	0.0823
			20	60.124	60.53	60.64	60.591	0.7710	0.1009	0.0807
2.07	0.50	0.784	100	38.682	37.673	37.402	37.278	3.7668	1.0601	0.3331
			20	50.659	51.164	51.299	51.361	1.3670	0.3837	0.1209
	1.00	0.674	100	47.899	46.575	46.21	46.059	3.9957	1.1211	0.3286
			20	72.102	73.425	73.79	73.941	2.4876	0.6983	0.2047
5.03	0.50	0.958	100	26.355	24.217	23.594	23.371	12.7689	3.6207	0.9550
			20	56.823	57.892	58.203	58.315	2.5578	0.7247	0.1914
	1.00	0.834	100	38.406	34.927	33.725	33.267	15.4477	4.9899	1.3767
			20	81.592	85.073	86.274	86.733	5.9274	1.9139	0.5292

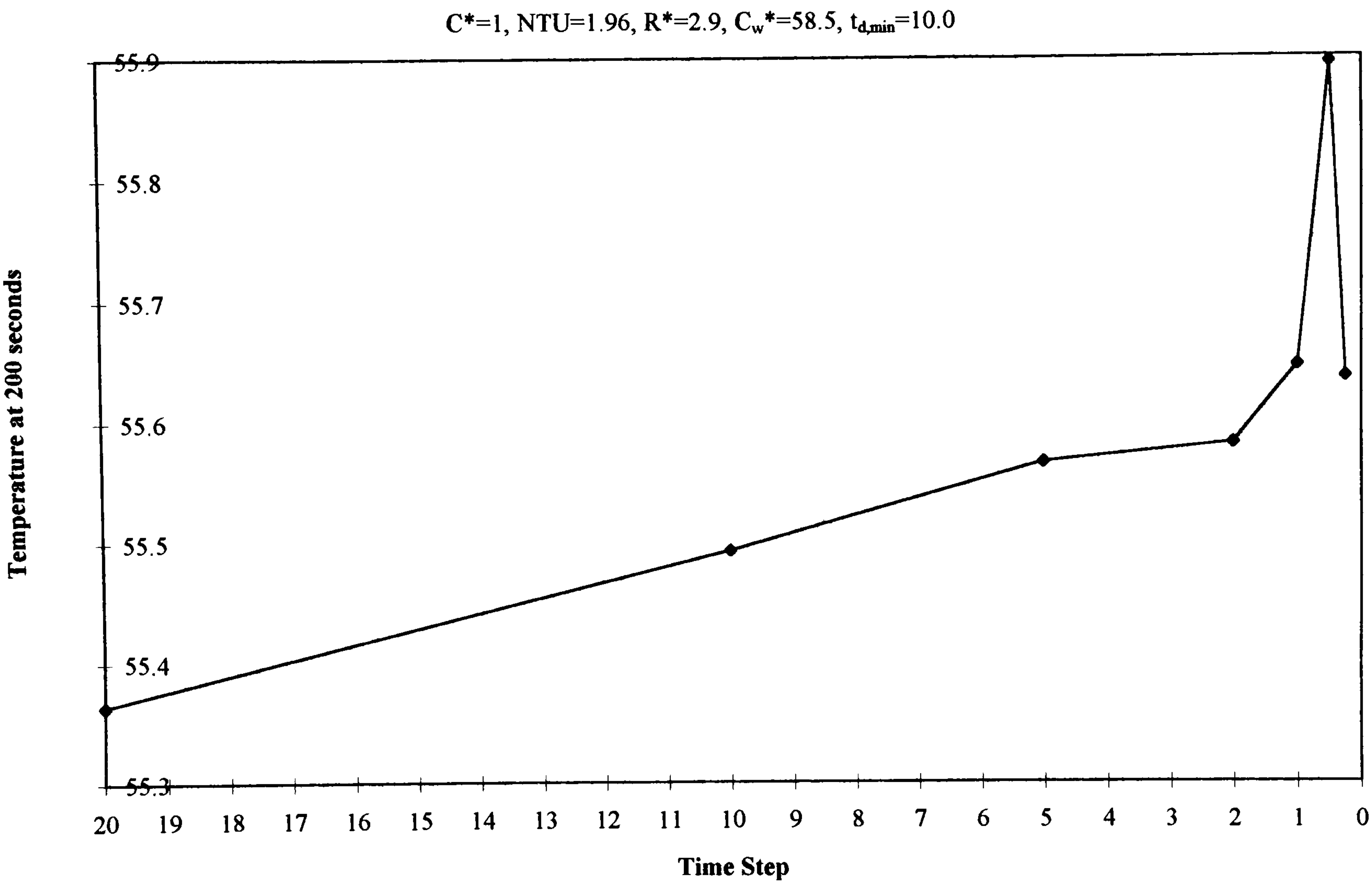


Figure 4.2 Temperature vs. Time Step

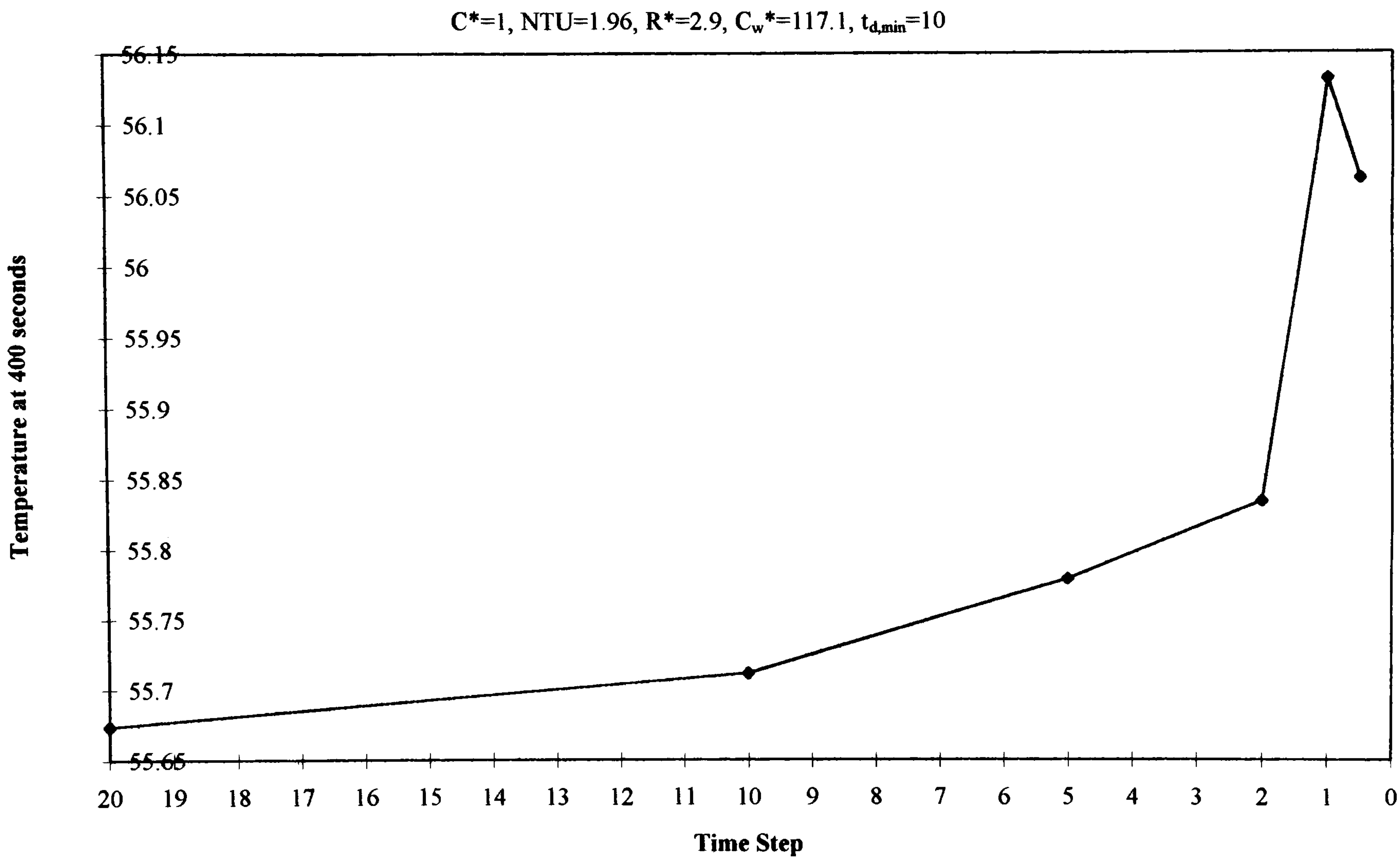


Figure 4.3 Temperature vs. Time Step

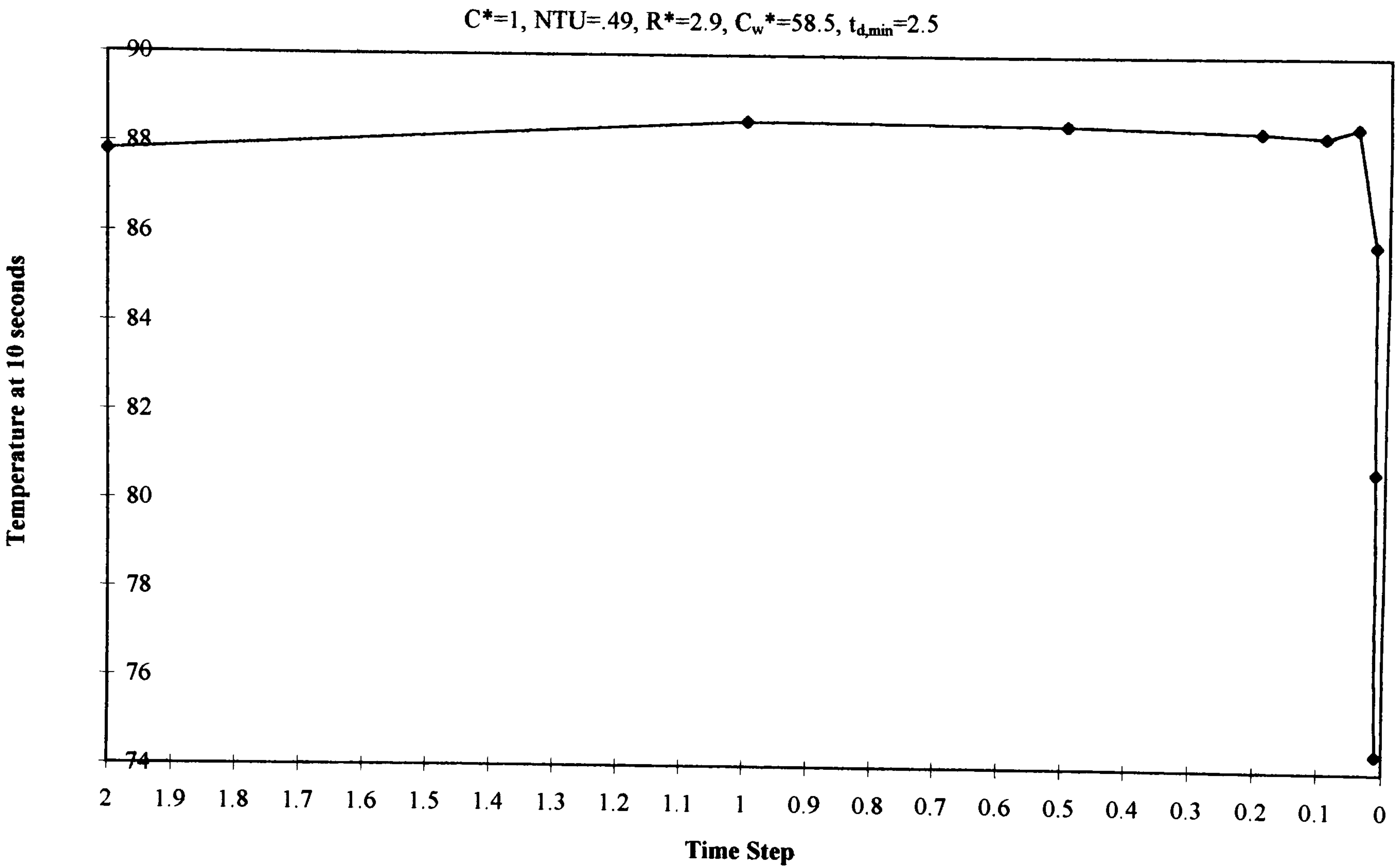


Figure 4.4 Temperature vs. Time Step

5. Validation of Thermal Network Solver Accuracy

Thermonet results were compared with five existing solutions found in literature for transient performance of heat exchangers. These solutions are briefly summarized in Table 5.1 below. The solutions were chosen for purpose of comparison because they cover a variety of operating parameters as well as solution methods.

Table 5.1 Comparison Summary

Author	Application	Solution Method	Comparison figure #	Stepped Fluid Mean Difference	Unstepped Fluid Mean Difference
London et al. (1964)	Counterflow $C^* = 1$	Electromechanical analog	5.1 5.2	9.09% 3.29%	1.81% 2.65%
Romie (1984)	Counterflow Start-up conditions	Finite Difference	5.3 5.4	0.66% 0.69%	1.80% 1.54%
Rizika (1956)	$C^* = 0$ $0 \le t^* \le 1$ Step change in C_{max} fluid	Analytical	5.5 5.6	1.74% 0.81%	— —
Myers et al. (1970)	$C^* = 0$ Step change in C_{max} fluid	Finite Difference	5.7 5.8	— —	0.19% 0.23%
Myers et al. (1967)	$C^* = 0$ Step change in C_{min} fluid	Analytical	5.9 5.10	3.18% 1.84%	— —

The percent mean difference was calculated as follows:

$$\frac{\text{average difference between solutions over the time domain}}{\text{total change in fluid outlet temp. from time } t = 0 \text{ to } t = \infty} \times 100\%$$

As can been seen in Table 5.1 the largest mean difference is for the comparison with the solution by London (1964). London's solution was derived by experimental techniques (electromechanical analog) and may have some error associated with it. The comparison with analytical solutions is more relevant and should be used as a test for validity. Two analytical solutions were utilized for comparison. The results from the solution by Rizika (1956) were calculated directly from his analytical expression. Thermonet gives results within 1.74% of the solution by Rizika. Results from Myers et al. (1967) were calculated from a graphical representation of his analytical solution. Thermonet gives results within 3.18% of the solution by Myers. Part of this difference could be attributed to inaccuracies in the graphical representation of his solution.

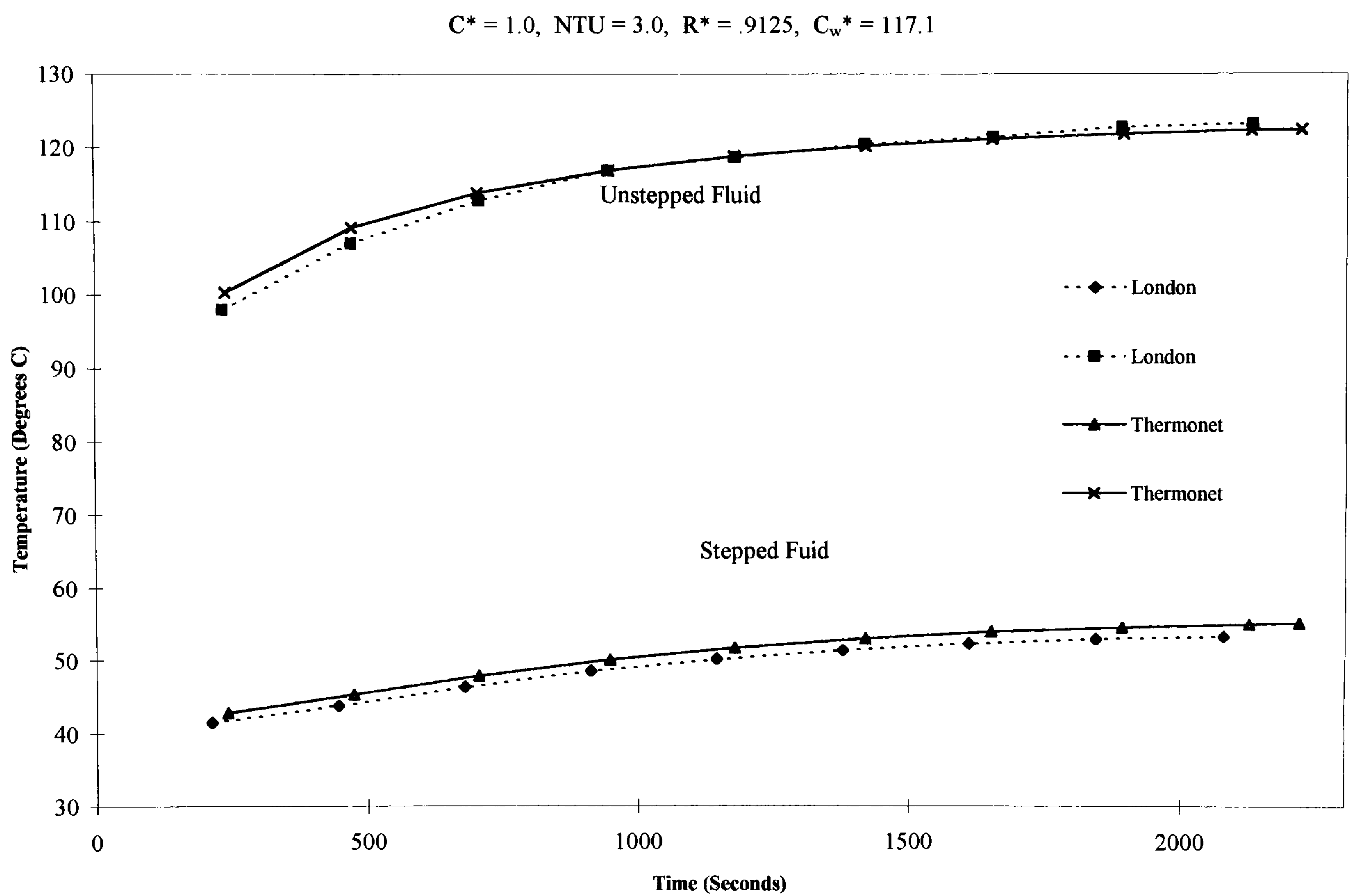


Figure 5.1 Comparison of Thermonet solution to solution by London et al. (1964)

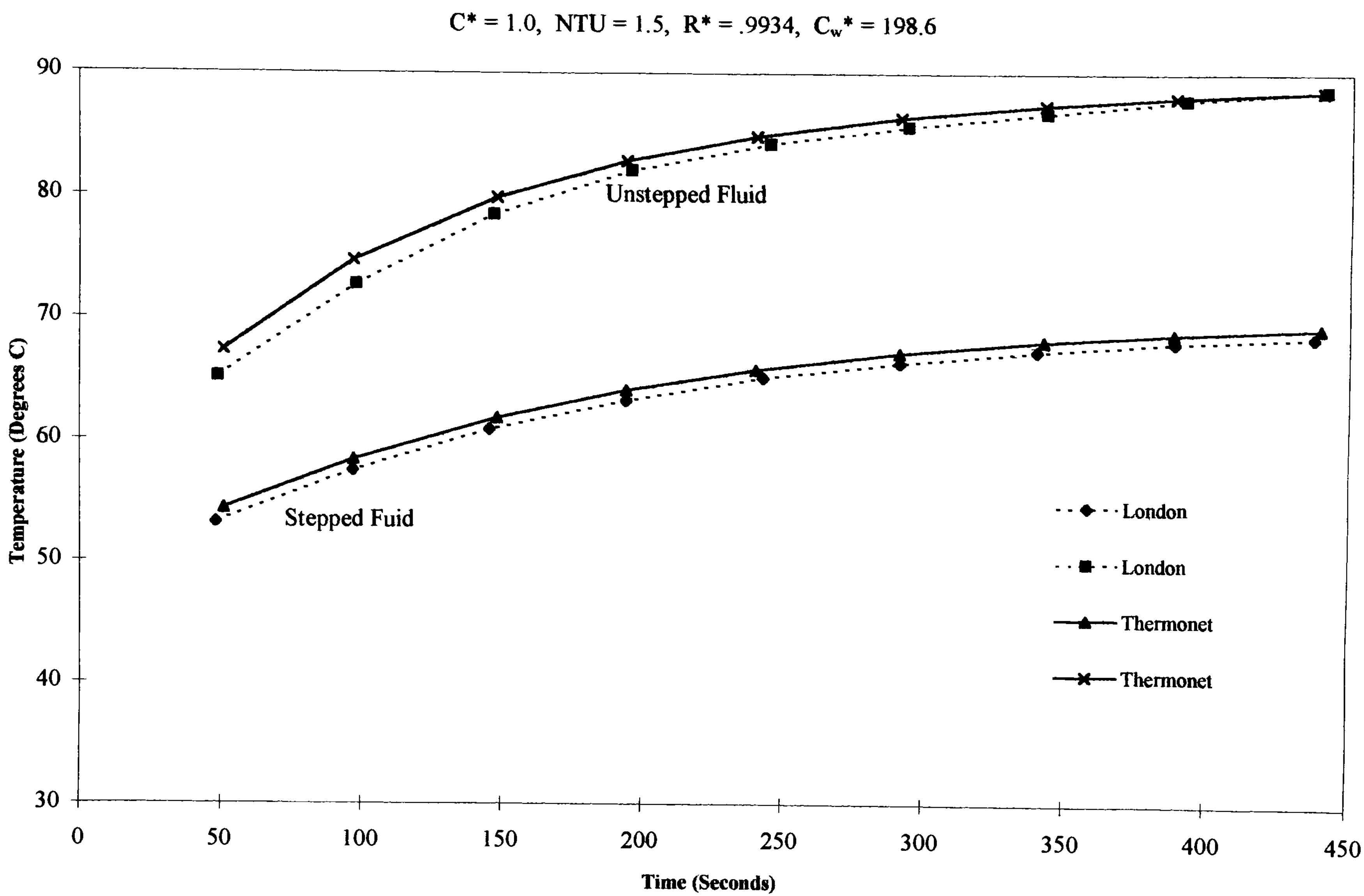


Figure 5.2 Comparison of Thermonet solution to solution by London et al. (1964)

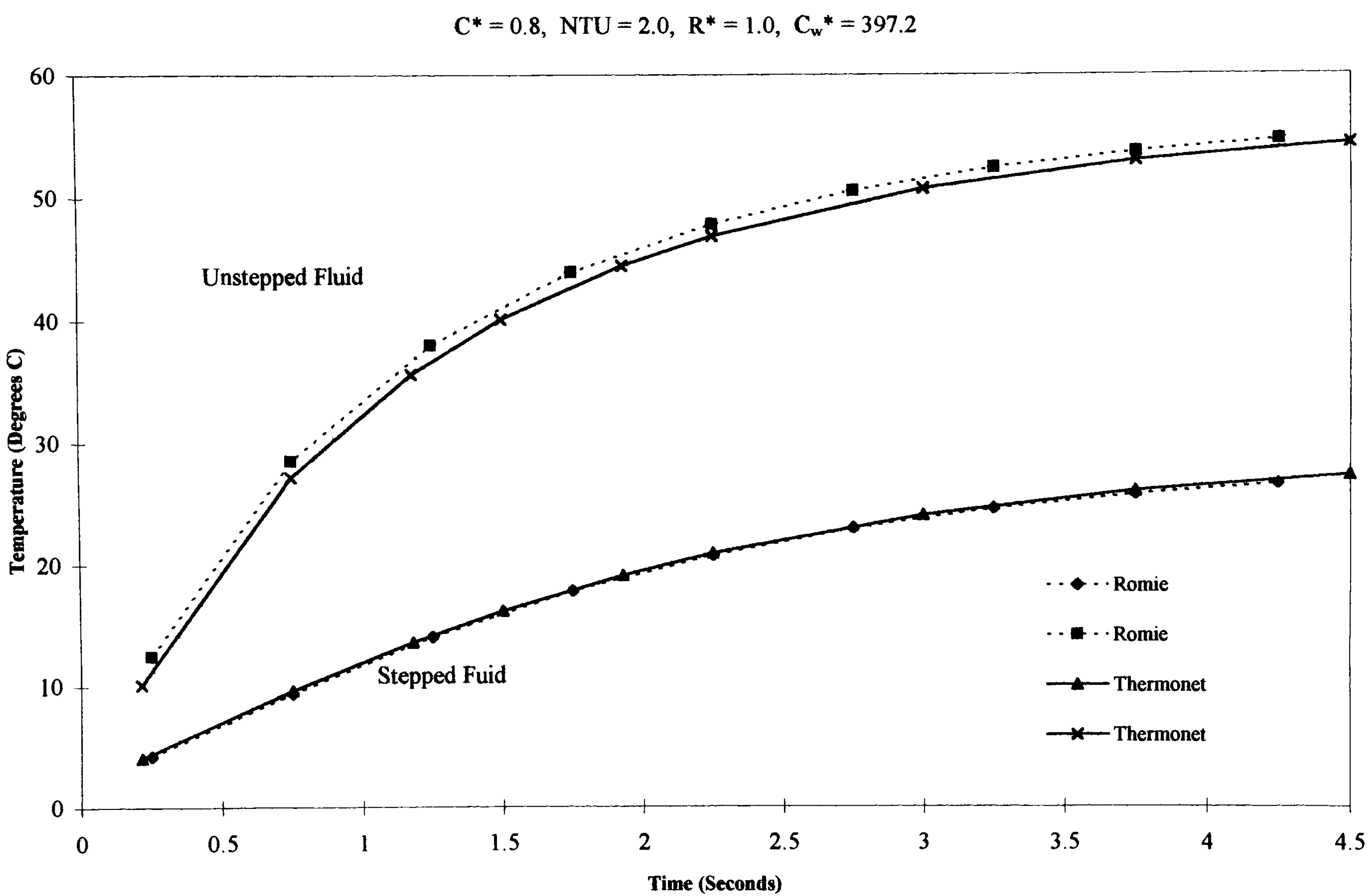


Figure 5.3 Comparison of Thermonet solution to solution by Romie (1984)

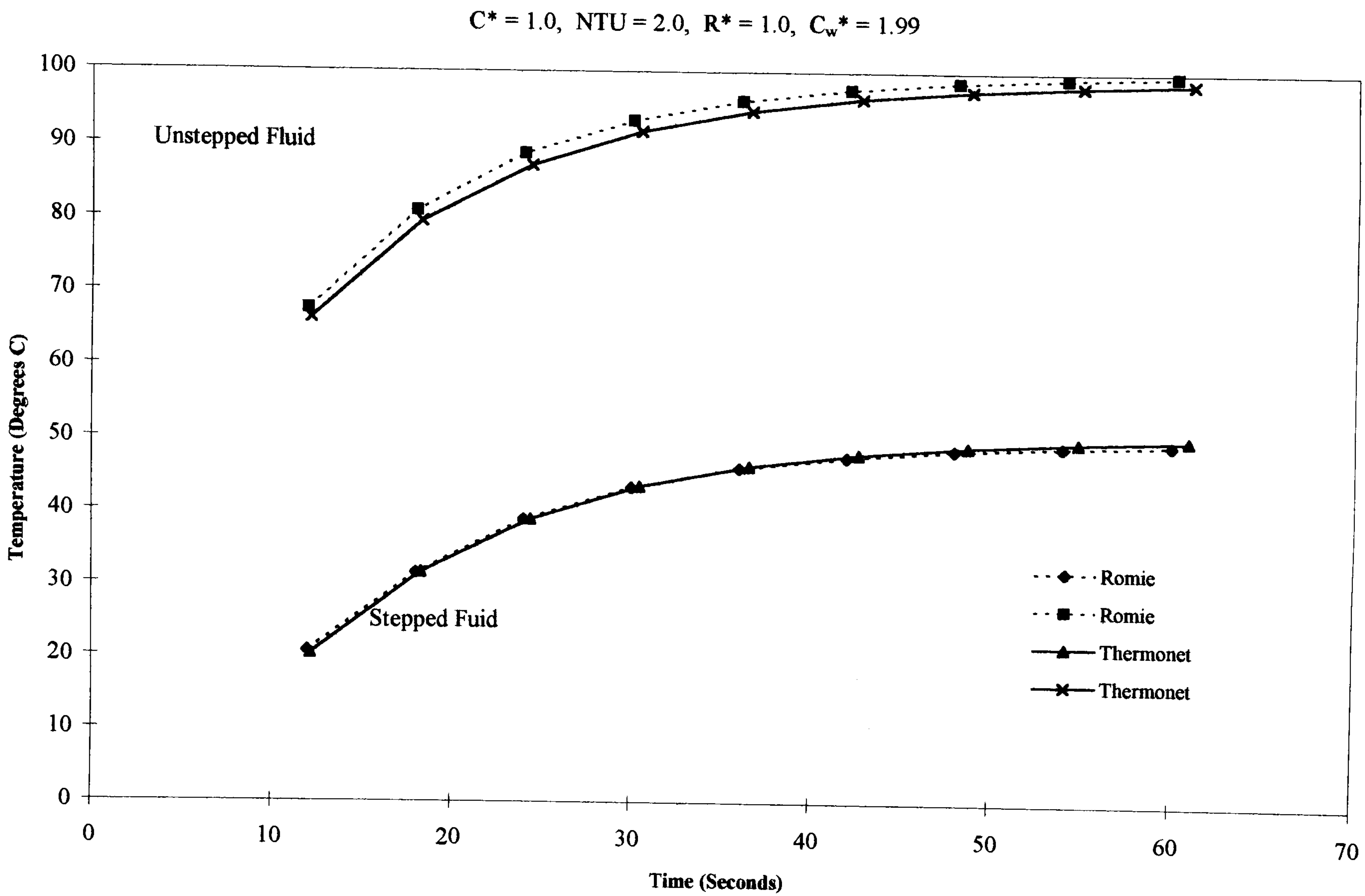


Figure 5.4 Comparison of Thermonet solution to solution by Romie (1984)

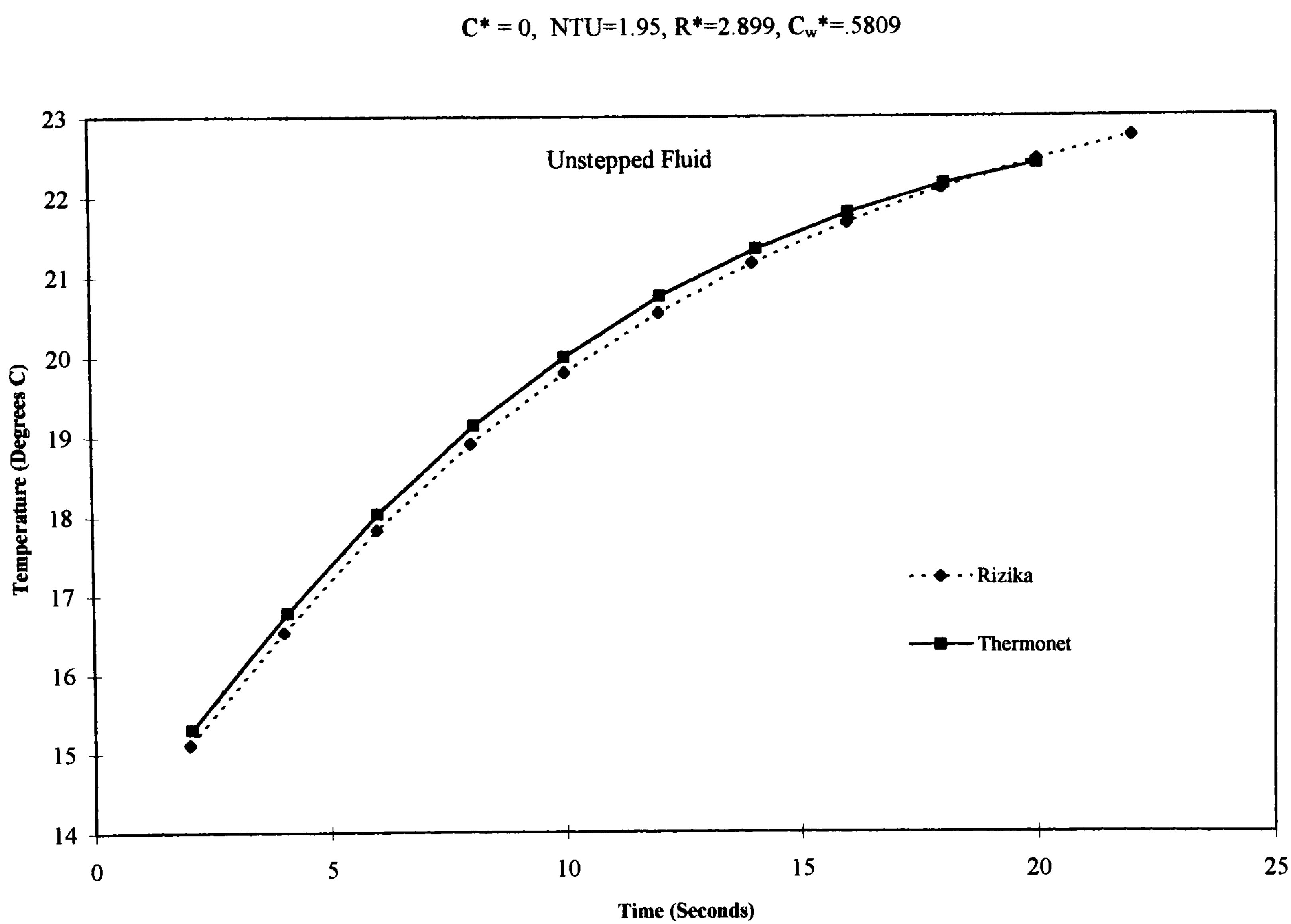


Figure 5.5 Comparison of Thermonet solution to solution by Rizika (1956)

$C^* = 0$, $NTU=1.55$, $R^*=1.08$, $C_w^*=.8781$

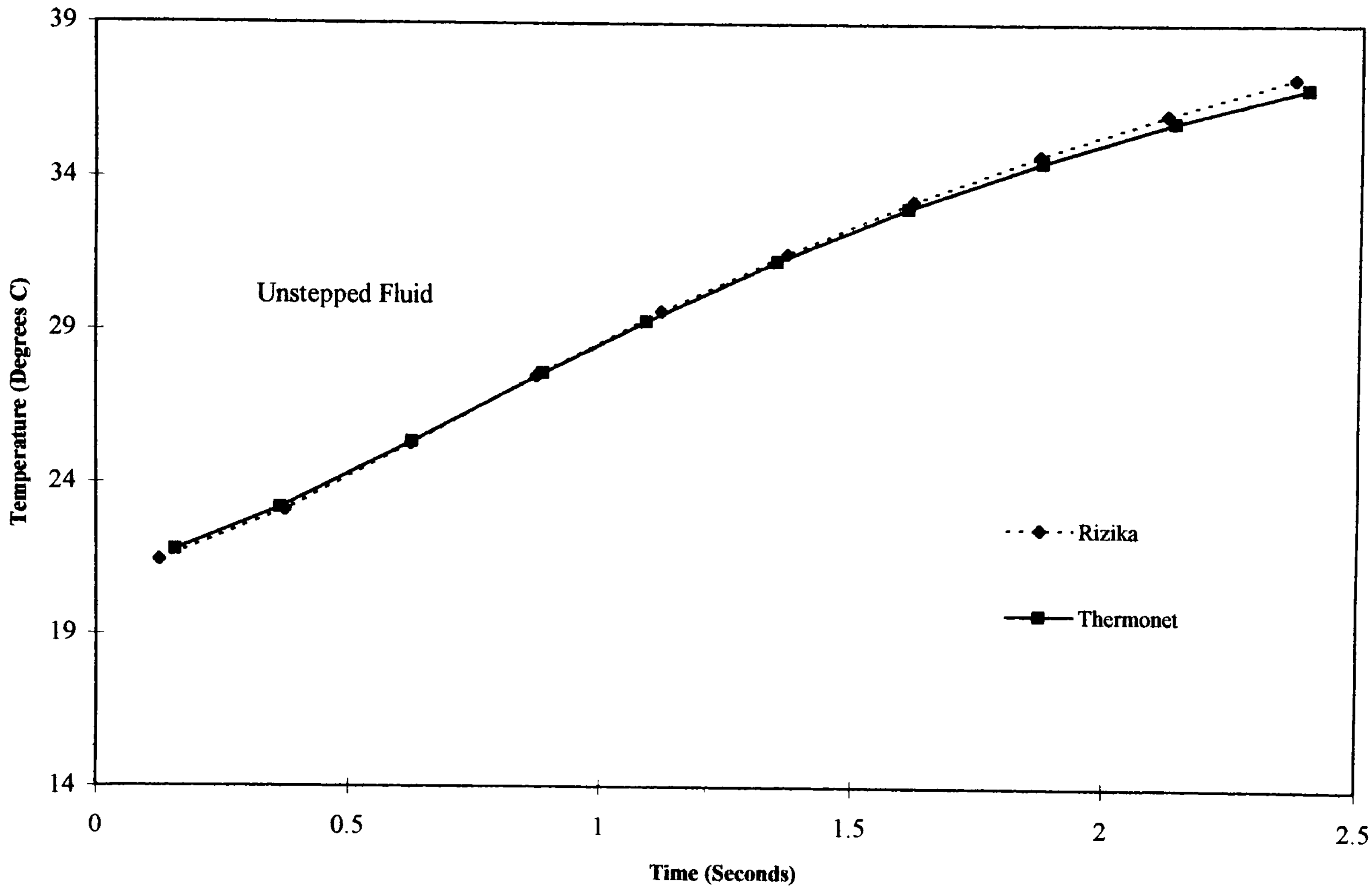


Figure 5.6 Comparison of Thermonet solution to solution by Rizika (1956)

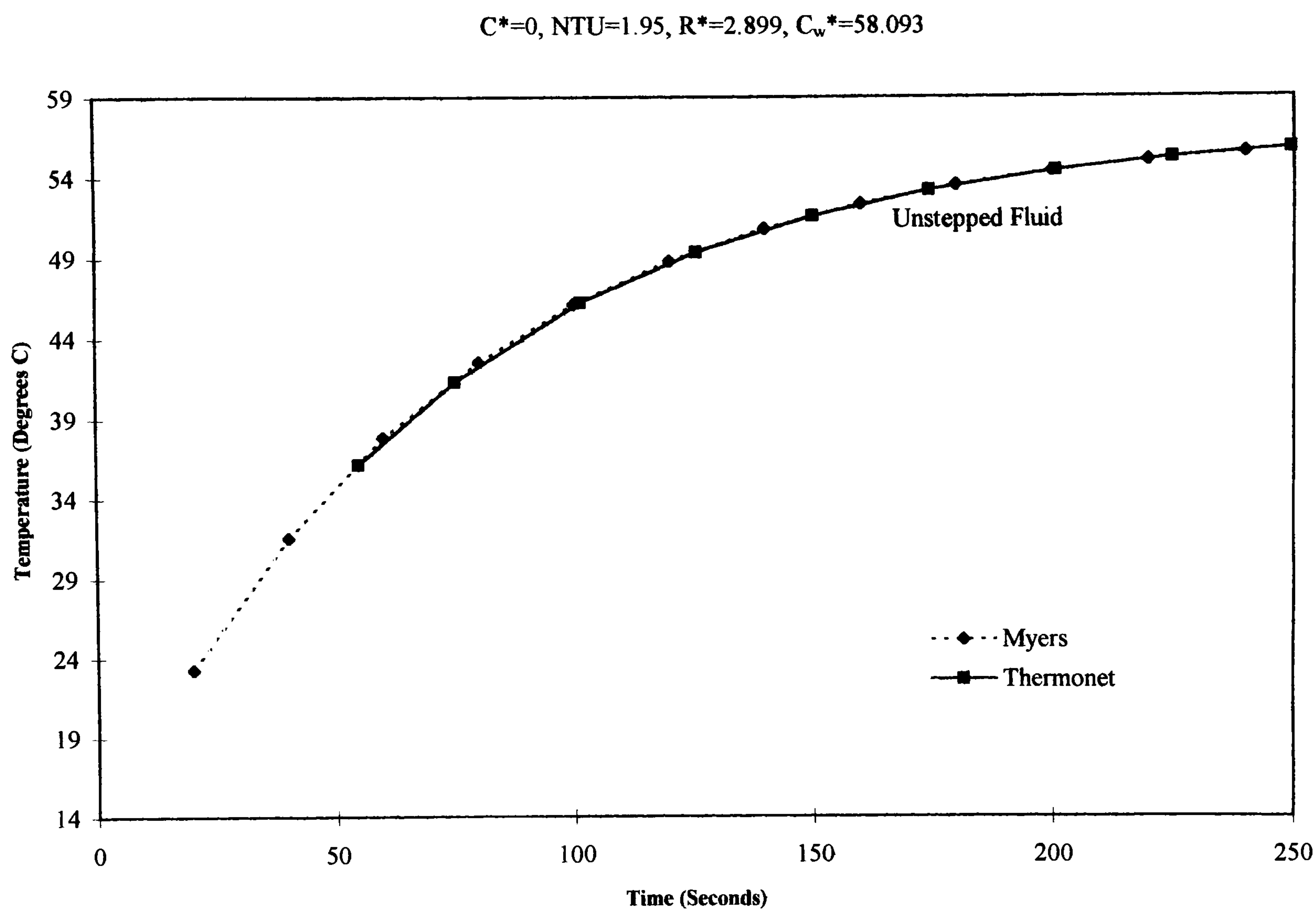


Figure 5.7 Comparison of Thermonet solution to solution by Myers et al. (1970)

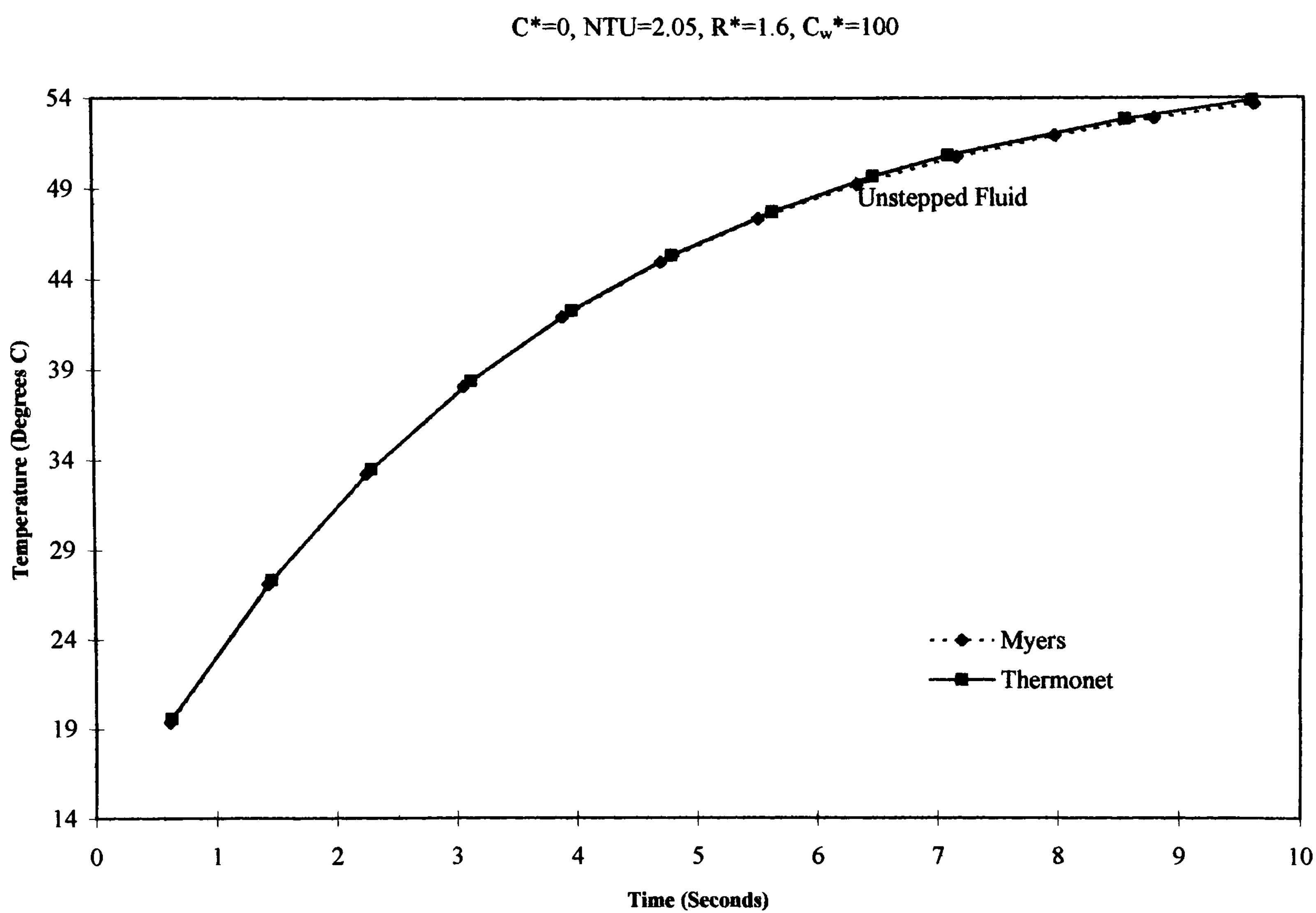


Figure 5.8 Comparison of Thermonet solution to solution by Myers et al. (1970)

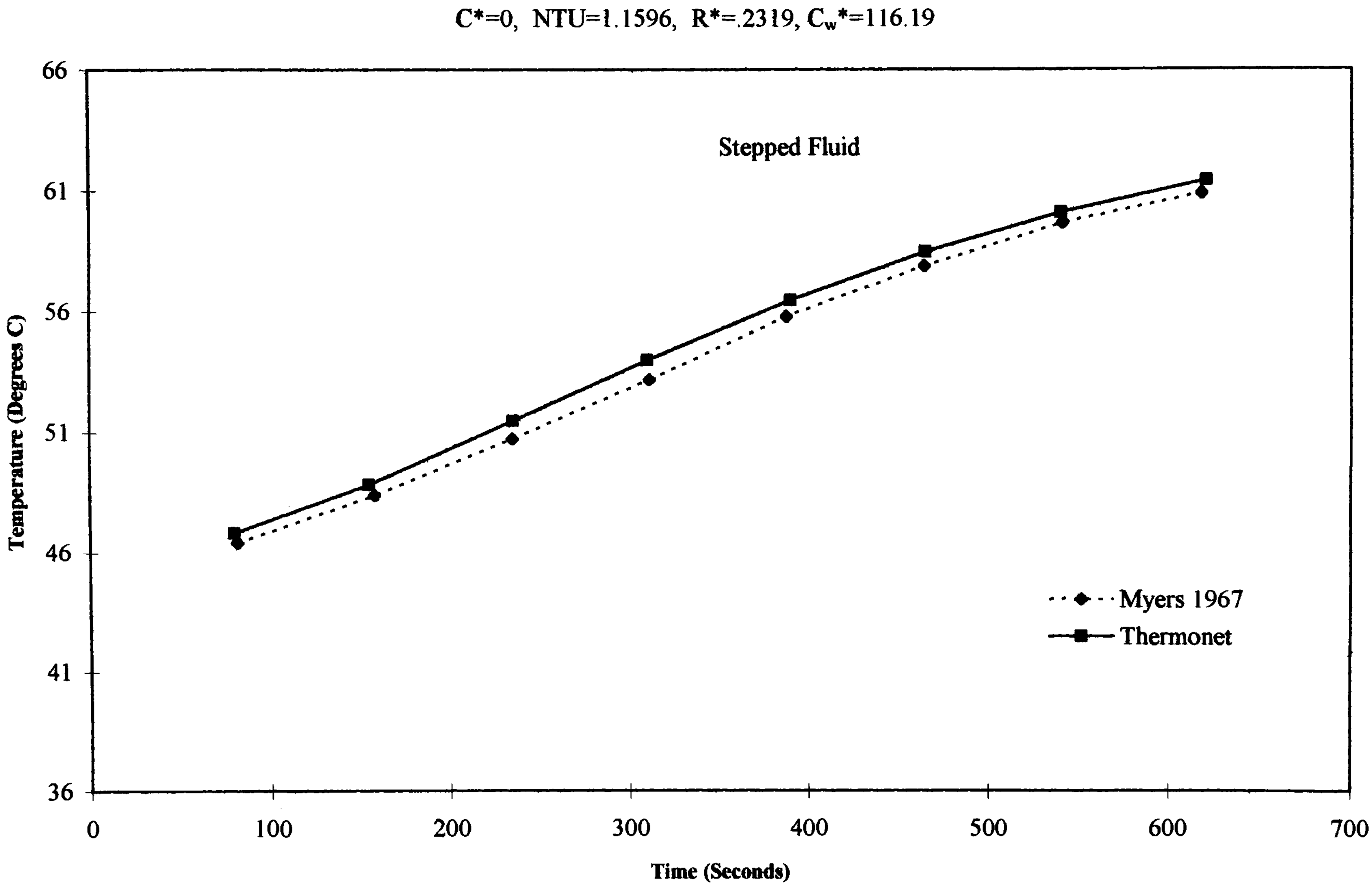


Figure 5.9 Comparison of Thermonet solution to solution by Myers et al. (1967)

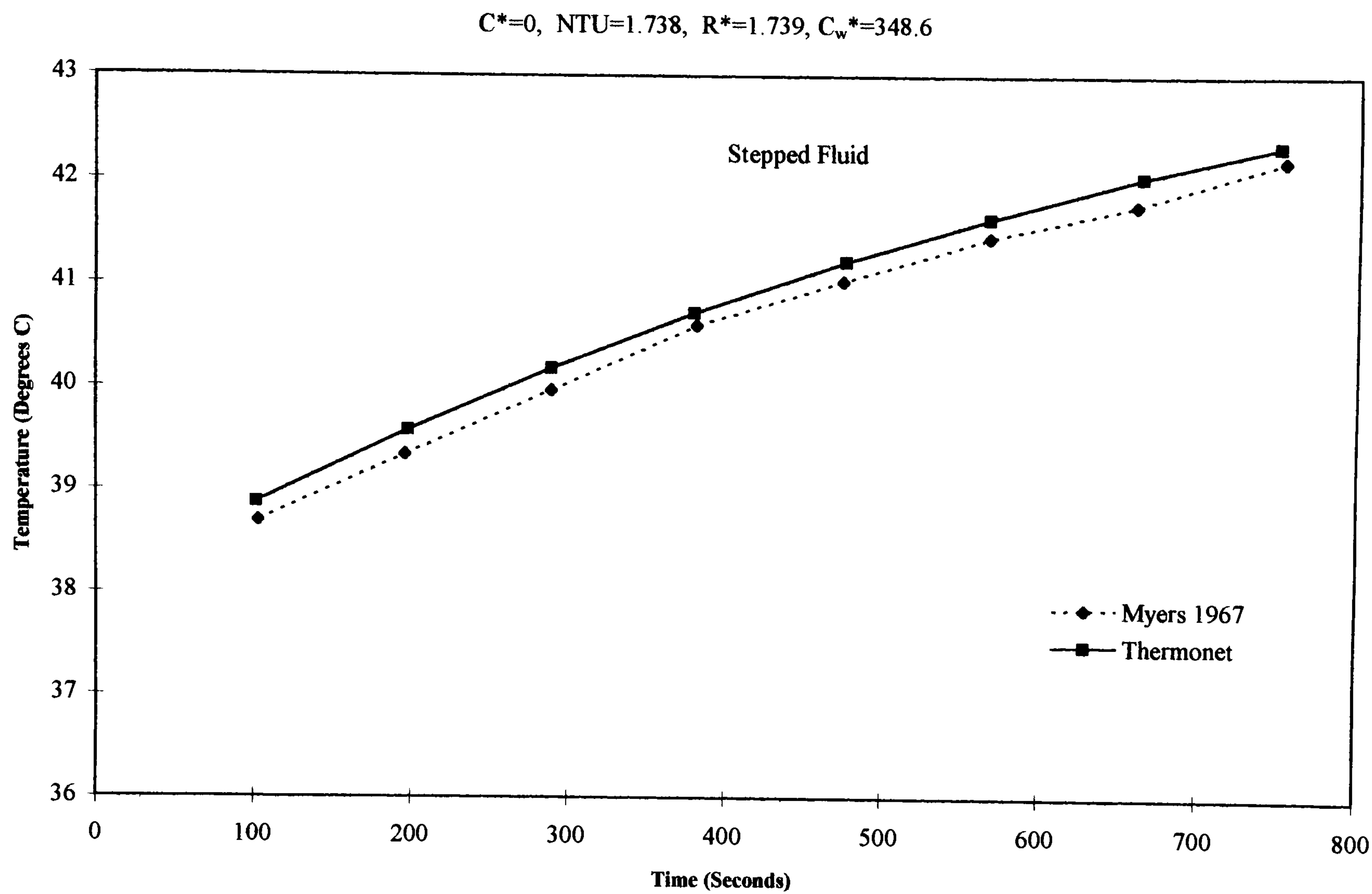


Figure 5.10 Comparison of Thermonet solution to solution by Myers et al. (1967)

6. Results for Counterflow Heat Exchangers

It has been previously noted that even though a large number of solutions exist to the transient heat exchanger problem, only a few are likely to be utilized in a design situation due to the complexity of some solutions. Of the four heat exchanger configurations previously discussed, configurations with $C^* = 0$ currently have the most useful solutions available. Solutions for this configuration cover a wide range of parameters and can be easily calculated. The same cannot be said for parallel flow, counterflow, and crossflow configurations with $C^* \neq 0$. Due to the thermal advantages and the common application of counterflow heat exchangers in engineering practice, the availability of a solution that covers a wide range of variables and that can be conveniently utilized would be very useful. The solutions currently available in literature are either too restrictive or too complicated to utilize (see section 3 for a complete discussion of available solutions). Transient solutions valid for counterflow configurations will be generated using Thermonet computer software and the results will be presented in tabular form. A tabular scheme is developed to present solutions covering the following ranges of parameters:

Dimensionless Parameter	Values of Parameters for Table Generation					
NTU	0.5	1.0	3.0			
C^*	0.2	0.6	1.0			
R^*	0.5	1.0	2.0			
C_w^*	1.0	10.0	50.0	100.0	400.0	1000.0
t_d^*	0.25	1.0	4.0			

The range of parameters was chosen to cover many practical heat exchanger applications. In order to present every possible combination of the above parameters , a large number of tables would be required. Fortunately, some simplifications can be made. Cima and London (1958) stated that transient solutions are virtually insensitive to variations in t_d^* for $C^* = 1$ and $\overline{C_w}^* > 10$. To verify this, transient solutions were calculated from Thermonet using two different values of t_d^* while all other parameters remained the same. The values of t_d^* used were 0.2 and 4.0. This represents a change in t_d^* by a factor of 16. The results of these comparisons are summarized below (actual comparisons can be found in Appendix C):

- Solutions for $\overline{C_w}^* = 50.0$ are virtually insensitive to changes in t_d^* for values of C^* ranging from 0.25 to 1.0. There is less than a 1% difference in the two solutions over the vast majority of the solution domain. The largest deviation of 2.3% occurs in the unstepped fluid at $C^* = 1$ shortly after the time step is imposed.

- Solutions for $\overline{C_w}^* = 10.0$ are more dependent on variations in t_d^* There is over a 3% difference in the two solutions over a large portion of the solution domain.

The above results greatly reduce the number of tables needed to present solution for the above parameters. Solutions for values of $\overline{C_w}^* \geq 50$ will have very little dependence on t_d^*. An intermediate value of $t_d^* = 1.0$ was used in the generation of the solutions to minimize the largest possible deviation to approximately 1%. Solutions for values of

$\overline{C_w}^*$ = 10 are presented for two separate ranges of t_d* This minimizes the largest possible deviation to approximately 2%. The solutions generated utilizing Thermonet are reported in Tables 6.1 through 6.3. Only solutions for $\overline{C_w}$ * = 1.0 are presented, generation of solutions for other values of $\overline{C_w}^*$ is part ongoing work at RIT.

Table 6.1 Results for NTU = 0.5 $C_w^* = 1.0$

NTU = 0.5 $C_w^* = 1.0$

t_d^*=.25	C* = 0.2						C* = 0.6						C* = 1.0					
	R* = .5		R* = 1		R* = 2		R* = .5		R* = 1		R* = 2		R* = .5		R* = 1		R* = 2	
t*	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$
0.7	0.16	0.08	0.20	0.08	0.23	0.09	0.14	0.08	0.17	0.08	0.20	0.09	0.14	0.09	0.18	0.09	0.21	0.09
1.4	0.53	0.22	0.62	0.21	0.69	0.23	0.53	0.22	0.63	0.22	0.70	0.23	0.50	0.23	0.58	0.23	0.65	0.24
2.1	0.79	0.36	0.85	0.35	0.91	0.37	0.78	0.37	0.85	0.36	0.90	0.37	0.74	0.38	0.81	0.37	0.86	0.38
2.8	0.91	0.49	0.94	0.49	0.97	0.51	0.89	0.50	0.92	0.49	0.95	0.51	0.86	0.50	0.89	0.50	0.92	0.51
3.5	0.96	0.61	0.97	0.61	0.99	0.63	0.94	0.61	0.96	0.61	0.97	0.63	0.92	0.62	0.93	0.61	0.95	0.62
4.2	0.98	0.71	0.99	0.71	0.99	0.74	0.97	0.72	0.97	0.72	0.98	0.73	0.95	0.72	0.96	0.71	0.97	0.72
4.9	0.99	0.80	0.99	0.80	1.00	0.82	0.98	0.80	0.98	0.80	0.99	0.81	0.97	0.80	0.97	0.79	0.98	0.80
5.6	1.00	0.87	1.00	0.87	1.00	0.89	0.99	0.87	0.99	0.87	0.99	0.88	0.98	0.86	0.98	0.86	0.98	0.86
6.3	1.00	0.92	1.00	0.92	1.00	0.93	0.99	0.91	0.99	0.92	1.00	0.92	0.99	0.90	0.99	0.90	0.99	0.90
7.0	1.00	0.95	1.00	0.95	1.00	0.96	1.00	0.95	1.00	0.95	1.00	0.95	0.99	0.94	0.99	0.93	0.99	0.94
7.7	1.00	0.97	1.00	0.97	1.00	0.98	1.00	0.97	1.00	0.97	1.00	0.97	1.00	0.96	1.00	0.96	1.00	0.96
8.4	1.00	0.99	1.00	0.98	1.00	0.99	1.00	0.99	1.00	0.98	1.00	0.98	1.00	0.97	1.00	0.97	1.00	0.97

t_d^*=1.0	C* = 0.2						C* = 0.6						C* = 1.0					
	R* = .5		R* = 1		R* = 2		R* = .5		R* = 1		R* = 2		R* = .5		R* = 1		R* = 2	
t*	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$
0.5	0.06	0.12	0.08	0.12	0.09	0.13	0.06	0.12	0.08	0.12	0.09	0.13	0.06	0.13	0.07	0.12	0.09	0.13
1.0	0.32	0.31	0.39	0.31	0.45	0.33	0.31	0.32	0.38	0.31	0.43	0.33	0.30	0.32	0.37	0.31	0.42	0.33
1.5	0.59	0.50	0.69	0.50	0.77	0.52	0.57	0.51	0.67	0.50	0.74	0.53	0.56	0.51	0.65	0.50	0.72	0.51
2.0	0.78	0.65	0.85	0.66	0.91	0.69	0.75	0.66	0.83	0.66	0.89	0.69	0.73	0.66	0.81	0.65	0.86	0.66
2.5	0.88	0.78	0.93	0.79	0.96	0.81	0.86	0.78	0.90	0.78	0.94	0.81	0.84	0.78	0.89	0.77	0.92	0.78
3.0	0.94	0.86	0.96	0.87	0.98	0.89	0.92	0.86	0.94	0.86	0.97	0.89	0.90	0.85	0.93	0.85	0.95	0.86
3.5	0.97	0.91	0.98	0.92	0.99	0.93	0.95	0.91	0.97	0.91	0.98	0.94	0.94	0.91	0.96	0.90	0.97	0.91
4.0	0.98	0.94	0.99	0.94	1.00	0.95	0.97	0.95	0.98	0.94	0.99	0.97	0.96	0.94	0.97	0.94	0.98	0.94
4.5	0.99	0.96	1.00	0.96	1.00	0.97	0.99	0.97	0.99	0.96	1.00	0.98	0.98	0.96	0.98	0.96	0.99	0.96
5.0	1.00	0.97	1.00	0.97	1.00	0.97	0.99	0.98	0.99	0.97	1.00	0.99	0.99	0.97	0.99	0.97	0.99	0.98
5.5	1.00	0.98	1.00	0.97	1.00	0.97	1.00	0.99	1.00	0.98	1.00	1.00	0.99	0.98	0.99	0.98	1.00	0.98
6.0	1.00	0.98	1.00	0.98	1.00	0.97	1.00	0.99	1.00	0.98	1.00	1.00	1.00	0.99	1.00	0.99	1.00	0.99

t_d^*=4.0	C* = 0.2						C* = 0.6						C* = 1.0					
	R* = .5		R* = 1		R* = 2		R* = .5		R* = 1		R* = 2		R* = .5		R* = 1		R* = 2	
t*	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$
0.4	0.03	0.14	0.04	0.13	0.05	0.15	0.03	0.14	0.04	0.14	0.05	0.15	0.03	0.15	0.04	0.14	0.05	0.15
0.8	0.20	0.34	0.25	0.34	0.29	0.37	0.19	0.34	0.24	0.34	0.28	0.36	0.19	0.35	0.24	0.34	0.27	0.35
1.2	0.44	0.53	0.53	0.54	0.60	0.58	0.42	0.53	0.51	0.53	0.58	0.55	0.41	0.53	0.50	0.52	0.56	0.53
1.6	0.64	0.69	0.74	0.70	0.81	0.74	0.62	0.68	0.71	0.68	0.78	0.70	0.60	0.67	0.69	0.66	0.76	0.68
2.0	0.78	0.80	0.85	0.81	0.91	0.84	0.76	0.79	0.83	0.79	0.89	0.81	0.74	0.78	0.81	0.77	0.87	0.78
2.4	0.86	0.87	0.92	0.88	0.96	0.91	0.84	0.86	0.90	0.86	0.94	0.88	0.83	0.85	0.88	0.84	0.92	0.85
2.8	0.92	0.92	0.95	0.92	0.98	0.94	0.90	0.91	0.94	0.91	0.96	0.92	0.89	0.90	0.92	0.89	0.95	0.90
3.2	0.95	0.95	0.97	0.95	0.99	0.97	0.94	0.94	0.96	0.94	0.98	0.95	0.92	0.93	0.95	0.93	0.97	0.93
3.6	0.97	0.96	0.98	0.97	0.99	0.98	0.96	0.96	0.97	0.96	0.99	0.96	0.95	0.95	0.96	0.95	0.98	0.95
4.0	0.98	0.98	0.99	0.98	1.00	0.98	0.98	0.97	0.98	0.97	0.99	0.97	0.97	0.96	0.98	0.96	0.99	0.97
4.4	0.99	0.98	1.00	0.98	1.00	0.99	0.99	0.98	0.99	0.98	0.99	0.98	0.98	0.97	0.98	0.97	0.99	0.98
4.8	1.00	0.99	1.00	0.99	1.00	0.99	0.99	0.98	0.99	0.98	1.00	0.98	0.99	0.98	0.99	0.98	0.99	0.98

Table 6.2 Results for NTU = 1.0 $C_w^* = 1.0$

NTU = 1.0 $C_w^* = 1.0$

$t_d^*=.25$	$C^* = 0.2$						$C^* = 0.6$						$C^* = 1.0$					
	$R^* = .5$		$R^* = 1$		$R^* = 2$		$R^* = .5$		$R^* = 1$		$R^* = 2$		$R^* = .5$		$R^* = 1$		$R^* = 2$	
t^*	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$
0.55	0.057	0.090	0.082	0.089	0.103	0.096	0.050	0.096	0.072	0.095	0.091	0.101	0.045	0.104	0.065	0.101	0.083	0.107
1.10	0.342	0.225	0.446	0.226	0.529	0.237	0.301	0.237	0.394	0.236	0.469	0.245	0.272	0.252	0.355	0.249	0.423	0.256
1.65	0.655	0.352	0.756	0.356	0.830	0.369	0.584	0.366	0.675	0.366	0.744	0.375	0.531	0.383	0.615	0.381	0.680	0.386
2.20	0.842	0.469	0.896	0.475	0.933	0.488	0.760	0.478	0.813	0.480	0.852	0.488	0.701	0.494	0.751	0.493	0.790	0.496
2.75	0.929	0.575	0.951	0.583	0.966	0.596	0.853	0.576	0.878	0.579	0.898	0.586	0.797	0.588	0.823	0.588	0.846	0.590
3.30	0.968	0.671	0.975	0.681	0.980	0.693	0.902	0.661	0.914	0.665	0.926	0.672	0.855	0.669	0.869	0.669	0.884	0.670
3.85	0.986	0.757	0.987	0.767	0.990	0.777	0.932	0.734	0.938	0.738	0.946	0.745	0.893	0.737	0.902	0.737	0.913	0.738
4.40	0.996	0.832	0.995	0.839	0.996	0.847	0.953	0.796	0.956	0.800	0.962	0.806	0.921	0.794	0.927	0.795	0.935	0.795
4.95	1.000	0.894	1.000	0.897	1.000	0.903	0.968	0.847	0.970	0.850	0.974	0.856	0.942	0.841	0.946	0.841	0.953	0.841
5.50	1.000	0.943	1.000	0.942	1.000	0.945	0.979	0.888	0.980	0.890	0.984	0.894	0.959	0.879	0.961	0.879	0.967	0.878
6.05	1.000	0.979	1.000	0.974	1.000	0.976	0.988	0.919	0.988	0.920	0.991	0.924	0.971	0.908	0.973	0.908	0.977	0.907
6.60	1.000	1.000	1.000	0.998	1.000	0.998	0.994	0.942	0.994	0.943	0.996	0.946	0.981	0.930	0.982	0.930	0.985	0.930

$t_d^*=1.0$	$C^* = 0.2$						$C^* = 0.6$						$C^* = 1.0$					
	$R^* = .5$		$R^* = 1$		$R^* = 2$		$R^* = .5$		$R^* = 1$		$R^* = 2$		$R^* = .5$		$R^* = 1$		$R^* = 2$	
t^*	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$
0.40	0.021	0.142	0.030	0.141	0.038	0.154	0.019	0.149	0.028	0.145	0.034	0.155	0.017	0.157	0.025	0.152	0.032	0.160
0.80	0.160	0.335	0.224	0.341	0.277	0.364	0.142	0.347	0.200	0.345	0.248	0.360	0.128	0.358	0.181	0.353	0.225	0.361
1.20	0.402	0.504	0.513	0.518	0.601	0.543	0.361	0.516	0.459	0.517	0.543	0.535	0.328	0.522	0.420	0.520	0.496	0.525
1.60	0.630	0.644	0.736	0.661	0.815	0.685	0.572	0.650	0.668	0.654	0.748	0.673	0.526	0.650	0.617	0.650	0.692	0.653
2.00	0.790	0.754	0.864	0.773	0.915	0.792	0.729	0.755	0.798	0.759	0.857	0.778	0.679	0.748	0.749	0.749	0.805	0.752
2.40	0.887	0.833	0.930	0.853	0.957	0.863	0.832	0.832	0.875	0.835	0.915	0.854	0.786	0.821	0.832	0.822	0.872	0.825
2.80	0.940	0.886	0.963	0.903	0.976	0.905	0.896	0.887	0.921	0.888	0.948	0.906	0.857	0.874	0.887	0.875	0.915	0.878
3.20	0.969	0.920	0.981	0.932	0.986	0.928	0.936	0.924	0.950	0.924	0.969	0.940	0.904	0.911	0.924	0.912	0.945	0.915
3.60	0.985	0.941	0.990	0.950	0.991	0.940	0.961	0.947	0.968	0.947	0.983	0.962	0.936	0.937	0.949	0.938	0.965	0.941
4.00	0.994	0.955	0.995	0.959	0.994	0.946	0.977	0.963	0.980	0.962	0.992	0.975	0.957	0.954	0.967	0.955	0.979	0.959
4.40	0.998	0.964	0.997	0.962	0.995	0.951	0.987	0.973	0.987	0.971	0.997	0.984	0.972	0.966	0.979	0.968	0.988	0.971
4.80	1.000	0.966	0.998	0.962	0.996	0.955	0.994	0.980	0.992	0.977	1.000	0.989	0.982	0.974	0.987	0.976	0.995	0.979

$t_d^*=4.0$	$C^* = 0.2$						$C^* = 0.6$						$C^* = 1.0$					
	$R^* = .5$		$R^* = 1$		$R^* = 2$		$R^* = .5$		$R^* = 1$		$R^* = 2$		$R^* = .5$		$R^* = 1$		$R^* = 2$	
t^*	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$
0.30	0.010	0.155	0.013	0.156	0.016	0.172	0.009	0.160	0.012	0.159	0.015	0.170	0.008	0.166	0.011	0.160	0.015	0.167
0.60	0.072	0.353	0.104	0.361	0.130	0.390	0.063	0.356	0.092	0.359	0.116	0.376	0.057	0.361	0.083	0.355	0.106	0.362
0.90	0.216	0.527	0.296	0.545	0.362	0.582	0.192	0.522	0.264	0.531	0.323	0.549	0.174	0.522	0.240	0.518	0.294	0.523
1.20	0.404	0.667	0.516	0.691	0.605	0.729	0.362	0.652	0.464	0.665	0.546	0.683	0.332	0.646	0.424	0.644	0.501	0.647
1.50	0.582	0.773	0.695	0.798	0.781	0.832	0.529	0.752	0.633	0.765	0.715	0.781	0.488	0.740	0.584	0.739	0.662	0.742
1.80	0.725	0.849	0.817	0.871	0.885	0.898	0.667	0.825	0.755	0.836	0.823	0.850	0.622	0.810	0.704	0.810	0.772	0.812
2.10	0.827	0.900	0.893	0.917	0.940	0.938	0.771	0.876	0.837	0.886	0.889	0.897	0.727	0.862	0.790	0.860	0.844	0.863
2.40	0.895	0.934	0.939	0.947	0.969	0.961	0.845	0.912	0.892	0.919	0.928	0.928	0.805	0.898	0.851	0.898	0.891	0.900
2.70	0.939	0.956	0.966	0.964	0.985	0.973	0.897	0.937	0.928	0.942	0.953	0.949	0.862	0.925	0.895	0.924	0.924	0.927
3.00	0.966	0.969	0.982	0.975	0.993	0.981	0.932	0.954	0.952	0.957	0.969	0.962	0.903	0.944	0.925	0.943	0.947	0.945
3.30	0.983	0.977	0.992	0.981	0.998	0.985	0.955	0.965	0.968	0.967	0.980	0.970	0.932	0.958	0.947	0.957	0.963	0.958
3.60	0.993	0.982	0.997	0.985	1.000	0.987	0.971	0.973	0.979	0.974	0.987	0.976	0.953	0.967	0.963	0.967	0.975	0.967

Table 6.3 Results for NTU = 3.0 $C_w^* = 1.0$

NTU = 3.0 $C_w^* = 1.0$

t_d^*=.25	C* = 0.2						C* = 0.6						C* = 1.0					
	R* = .5		R* = 1		R* = 2		R* = .5		R* = 1		R* = 2		R* = 5		R* = 1		R* = 2	
t*	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$
0.6	0.058	0.237	0.077	0.240	0.101	0.248	0.033	0.240	0.049	0.239	0.058	0.248	0.021	0.256	0.030	0.256	0.039	0.259
1.2	0.306	0.444	0.391	0.452	0.466	0.460	0.172	0.436	0.225	0.437	0.270	0.445	0.112	0.452	0.148	0.452	0.181	0.454
1.8	0.619	0.594	0.689	0.603	0.740	0.611	0.376	0.570	0.429	0.572	0.473	0.579	0.261	0.579	0.304	0.579	0.339	0.580
2.4	0.808	0.703	0.837	0.713	0.855	0.720	0.538	0.666	0.575	0.668	0.605	0.674	0.399	0.666	0.435	0.667	0.463	0.668
3.0	0.901	0.784	0.908	0.794	0.913	0.800	0.652	0.738	0.677	0.739	0.698	0.745	0.511	0.731	0.539	0.731	0.563	0.732
3.6	0.952	0.846	0.951	0.855	0.951	0.859	0.736	0.792	0.754	0.794	0.769	0.799	0.602	0.780	0.625	0.780	0.644	0.781
4.2	0.985	0.892	0.980	0.900	0.976	0.903	0.800	0.835	0.813	0.837	0.824	0.842	0.677	0.819	0.695	0.819	0.711	0.819
4.8	1.000	0.927	1.000	0.934	0.994	0.935	0.850	0.870	0.860	0.871	0.868	0.875	0.738	0.850	0.752	0.850	0.765	0.851
5.4	1.000	0.952	1.000	0.958	1.000	0.958	0.890	0.897	0.896	0.898	0.901	0.902	0.788	0.876	0.800	0.876	0.811	0.876
6.0	1.000	0.970	1.000	0.975	1.000	0.974	0.920	0.919	0.925	0.919	0.928	0.923	0.829	0.897	0.839	0.897	0.848	0.897
6.6	1.000	0.982	1.000	0.987	1.000	0.985	0.945	0.936	0.947	0.936	0.949	0.940	0.863	0.915	0.871	0.915	0.879	0.915
7.2	1.000	0.991	1.000	0.995	1.000	0.993	0.964	0.950	0.965	0.950	0.965	0.953	0.892	0.930	0.898	0.929	0.904	0.929

t_d^*=1.0	C* = 0.2						C* = 0.6						C* = 1.0					
	R* = .5		R* = 1		R* = 2		R* = .5		R* = 1		R* = 2		R* = .5		R* = 1		R* = 2	
t*	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$
0.4	0.023	0.319	0.024	0.327	0.028	0.346	0.016	0.313	0.017	0.316	0.020	0.327	0.010	0.324	0.012	0.323	0.015	0.327
0.8	0.100	0.567	0.151	0.583	0.201	0.604	0.059	0.538	0.089	0.545	0.120	0.555	0.038	0.541	0.059	0.541	0.080	0.542
1.2	0.310	0.723	0.419	0.740	0.507	0.760	0.188	0.676	0.257	0.683	0.320	0.692	0.128	0.667	0.176	0.668	0.223	0.668
1.6	0.565	0.825	0.673	0.840	0.746	0.857	0.369	0.766	0.450	0.774	0.517	0.781	0.265	0.748	0.328	0.749	0.383	0.750
2.0	0.763	0.891	0.835	0.904	0.876	0.917	0.541	0.830	0.610	0.836	0.664	0.843	0.411	0.806	0.470	0.807	0.521	0.807
2.4	0.884	0.934	0.923	0.943	0.940	0.953	0.675	0.875	0.728	0.881	0.767	0.887	0.539	0.849	0.588	0.850	0.632	0.850
2.8	0.950	0.961	0.970	0.967	0.974	0.974	0.774	0.908	0.812	0.913	0.840	0.918	0.644	0.881	0.683	0.882	0.717	0.882
3.2	0.987	0.977	0.996	0.981	0.993	0.986	0.844	0.933	0.872	0.936	0.892	0.941	0.726	0.907	0.757	0.907	0.784	0.907
3.6	1.000	0.986	1.000	0.989	1.000	0.992	0.895	0.950	0.915	0.953	0.928	0.957	0.790	0.926	0.814	0.927	0.836	0.927
4.0	1.000	0.992	1.000	0.994	1.000	0.996	0.932	0.963	0.946	0.965	0.955	0.969	0.840	0.941	0.860	0.942	0.877	0.942
4.4	1.000	0.995	1.000	0.996	1.000	0.998	0.958	0.972	0.968	0.974	0.974	0.977	0.879	0.953	0.895	0.953	0.908	0.954
4.8	1.000	0.997	1.000	0.997	1.000	0.999	0.977	0.979	0.984	0.980	0.987	0.983	0.910	0.962	0.922	0.962	0.933	0.963

t_d^*=4.0	C* = 0.2						C* = 0.6						C* = 1.0					
	R* = .5		R* = 1		R* = 2		R* = 5		R* = 1		R* = 2		R* = .5		R* = 1		R* = 2	
t*	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$	$\varepsilon_{f,1}^*$	$\varepsilon_{f,2}^*$
0.4	0.020	0.427	0.022	0.444	0.025	0.472	0.012	0.407	0.014	0.413	0.017	0.426	0.008	0.409	0.011	0.408	0.020	0.394
0.8	0.090	0.688	0.149	0.713	0.201	0.741	0.056	0.639	0.088	0.650	0.122	0.662	0.038	0.627	0.060	0.628	0.096	0.617
1.2	0.312	0.830	0.424	0.851	0.519	0.873	0.194	0.767	0.270	0.777	0.341	0.788	0.137	0.745	0.192	0.746	0.249	0.738
1.6	0.577	0.908	0.689	0.922	0.769	0.938	0.392	0.846	0.483	0.854	0.560	0.863	0.294	0.818	0.365	0.819	0.422	0.813
2.0	0.781	0.950	0.856	0.959	0.900	0.969	0.580	0.896	0.658	0.903	0.719	0.911	0.460	0.868	0.527	0.868	0.573	0.864
2.4	0.903	0.972	0.941	0.977	0.958	0.983	0.723	0.929	0.780	0.934	0.824	0.941	0.602	0.903	0.656	0.903	0.690	0.900
2.8	0.965	0.982	0.981	0.985	0.983	0.989	0.822	0.951	0.860	0.955	0.890	0.960	0.713	0.928	0.753	0.928	0.779	0.925
3.2	0.995	0.988	1.000	0.989	0.994	0.992	0.887	0.965	0.913	0.968	0.932	0.972	0.796	0.946	0.825	0.946	0.843	0.944
3.6	1.000	0.990	1.000	0.991	0.999	0.993	0.930	0.974	0.947	0.976	0.958	0.980	0.856	0.959	0.877	0.959	0.890	0.957
4.0	1.000	0.992	1.000	0.992	1.000	0.994	0.959	0.980	0.969	0.982	0.975	0.985	0.900	0.968	0.915	0.968	0.924	0.967
4.4	1.000	0.993	1.000	0.993	1.000	0.994	0.977	0.984	0.983	0.985	0.986	0.988	0.932	0.975	0.943	0.975	0.949	0.974
4.8	1.000	0.993	1.000	0.993	1.000	0.994	0.989	0.987	0.993	0.988	0.993	0.990	0.955	0.980	0.962	0.980	0.967	0.979

7. Discussion of Results

Of the six dimensionless parameters (NTU, C*, R*, $\overline{C_w}^*$, t*, t_d*) needed to define a transient solution, the parameter which has the greatest effect on the length of time required to complete a transient is $\overline{C_w}^*$. Recall that the definition of $\overline{C_w}^*$ is the ratio of wall capacitance to capacitance of the C_{min} fluid:

$$\overline{C_w}^* = \overline{C_w} \;/\; \overline{C}_{min} = (Mc_p)_{wall}/(Mc_p)_{min}$$

Hence, the larger the value of $\overline{C_w}^*$, the larger the wall is in relation to the fluids. If the wall is large compared to the fluids, a large amount of time will be required to reach new steady state operating conditions because the thermal energy stored in the wall must be dissipated. Therefore, large values of $\overline{C_w}^*$ imply long time durations for the heat exchanger to reach new steady state operating conditions. Knowledge of this value will give the designer a general idea of the transient response of a heat exchanger.

It is demonstrated by comparison runs using Thermonet that the parameter t_d* has very little influence on transient solutions for $\overline{C_w}^* \geq 50$. The condition that exhibited the most dependence were for values of $\overline{C_w}^* = 50$, C* = 1.0, NTU = 0.5.

It can be seen in Table 6.1 for NTU = 0.5 that the value of C* has only a slight influence on the transient solution. As the value of NTU increases (Tables 6.2 and 6.3) C* has a greater effect.

8. Conclusions

Unlike steady state heat exchanger analysis, transient heat exchanger analysis is very complex. No generally accepted solution exists. Many authors have presented solutions in literature which are applicable only to limited ranges of parameters. An extensive literature survey has been presented in this thesis which has discussed major solutions available in literature. The application of solutions by Romie (1984) and Myers(1970) has been discussed in detail, presenting information not clearly discussed in the respective papers. Recommendations have been made regarding the application of solutions found in literature.

A thermal network solver software package (Thermonet) has been successfully utilized to model the transient response of heat exchangers. Thermonet solutions have been verified with five existing solutions found in literature.

Transient solutions valid for counterflow heat exchangers have been generated for $\overline{C_w}^* = 1.0$ and presented in a convenient tabular format. A tabular format has been developed to present solutions covering a wide range of independent parameters believed applicable to many practical applications. These solutions represent a significant contribution to the field of transient heat exchanger analysis for the following reasons:

- The solutions are valid for counterflow heat exchangers which are often utilized in industry due to their thermal advantages.

- Solutions cover a wider range of $\overline{C}_w^*$ values than the solution by Romie (1984). Solutions for the stepped fluid are valid for times less than one dwell time, the solution by Romie is not accurate in this range.
- Solutions are valid for a much wider range of parameters than the solution by London et al. (1964)
- Solutions are much more convenient to utilize than the analytical solution of Gvozdenac (1987)

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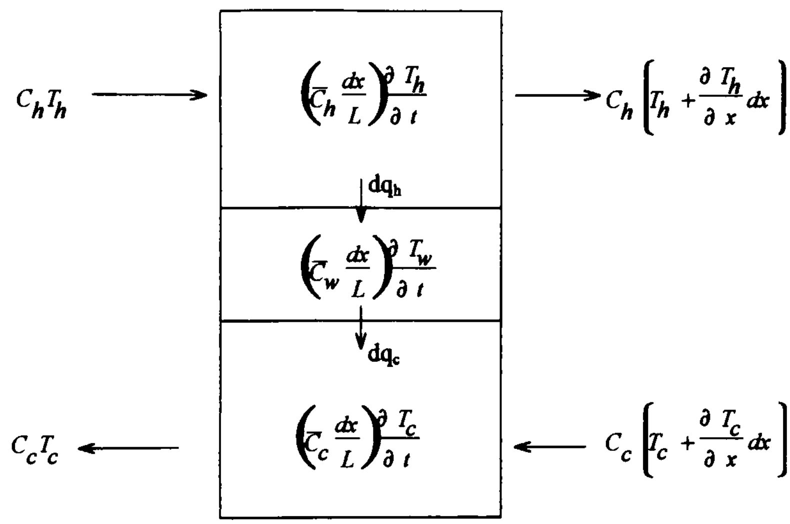
Appendices

Appendix A

Derivation of governing differential equations

The control volume shown in Figure A.1 shows all energy transfer terms associated with control volumes for the hot fluid, cold fluid, and wall. During its flow through the control volume, the hot fluid transfers heat to the wall by convection resulting in reduction of its outlet enthalpy and internal stored thermal energy.



$$dq_h = (\eta_o h)_h \left(\frac{A_h dx}{L}\right)(T_h - T_w)$$

$$dq_c = (\eta_o h)_c \left(\frac{A_c dx}{L}\right)(T_w - T_c)$$

Figure A.1 Incremental control volume for counterflow heat exchanger

Applying an energy balance for a control volume around the hot fluid yields:

$$(\text{Energy})_{in} - (\text{Energy})_{out} = (\text{Energy})_{stored}$$

$$C_h T_h - \left[C_h \left(T_h + \frac{\partial T_h}{\partial x} dx \right) \right] - dq_h = \left(\bar{C}_h \frac{dx}{L} \right) \frac{\partial T_h}{\partial t} \qquad \text{(A.1)}$$

$$C_h T_h - C_h T_h - C_h \frac{\partial T_h}{\partial x} dx - \left[(\eta_o h)_h \left(\frac{A_h dx}{L} \right) (T_h - T_w) \right] = \left(\bar{C}_h \frac{dx}{L} \right) \frac{\partial T_h}{\partial t} \qquad \text{(A.2)}$$

$$\left(\bar{C}_h \frac{dx}{L} \right) \frac{\partial T_h}{\partial t} + C_h \frac{\partial T_h}{\partial x} dx + C_h \frac{\partial T_h}{\partial x} dx = (\eta_o h)_h \left(\frac{A_h dx}{L} \right) (T_h - T_w) \qquad \text{(A.3)}$$

multiply both sides by L/dx

$$\bar{C}_h \frac{\partial T_h}{\partial t} + C_h L \frac{\partial T_h}{\partial x} = (\eta_o h A)_h (T_h - T_w) \qquad \text{(A.4)}$$

A similar energy balance for the cold fluid yields equation 2.4.

Applying an energy balance around the wall:

$$(\text{Energy})_{in} - (\text{Energy})_{out} = (\text{Energy})_{stored}$$

$$dq_h - dq_c = \left(\bar{C}_w \frac{dx}{L} \right) \frac{\partial T_w}{\partial t} \qquad \text{(A.5)}$$

$$(\eta_o h)_h \left(\frac{A_h dx}{L} \right) (T_h - T_w) - (\eta_o h)_c \left(\frac{A_c dx}{L} \right) (T_w - T_c) = \left(\bar{C}_w \frac{dx}{L} \right) \frac{\partial T_w}{\partial t} \qquad \text{(A.6)}$$

multiply both sides by L/dx

$$\bar{C}_w \frac{\partial T_w}{\partial t} - (\eta_o h A)_h (T_h - T_w) + (\eta_o h A)_h (T_h - T_w) = 0 \qquad \text{(A.7)}$$

Appendix B
Investigation of Solution by Myers et al.(1970)

The approximate solution proposed by Myers et al. (1970) is shown to be valid for all values of $\overline{C}_w^* > 1$. Table B.1 summarizes the values for each comparison figure that follows.

Table B.1

$\overline{C}_w^*$	NTU	R*	Fluids	Figure #
5.3	0.49	1.6	Water - Water	B.1
30.4	0.49	1.6	Water- Water	B.2
100.0	2.0	1.6	Air - Air	B.3
4000.0	2.0	1.6	Air -Air	B.4
10000.0	2.0	1.6	Air - Air	B.5
14020.0	2.0	1.6	Air -Air	B.6

All of the above solutions were generated with the following inlet temperatures:

	Temp of C_{min} fluid (Degrees C)	Temp of C_{max} fluid (Degrees C)
Before Step Input	100.0	0.0
After Step Input	100.0	50.0

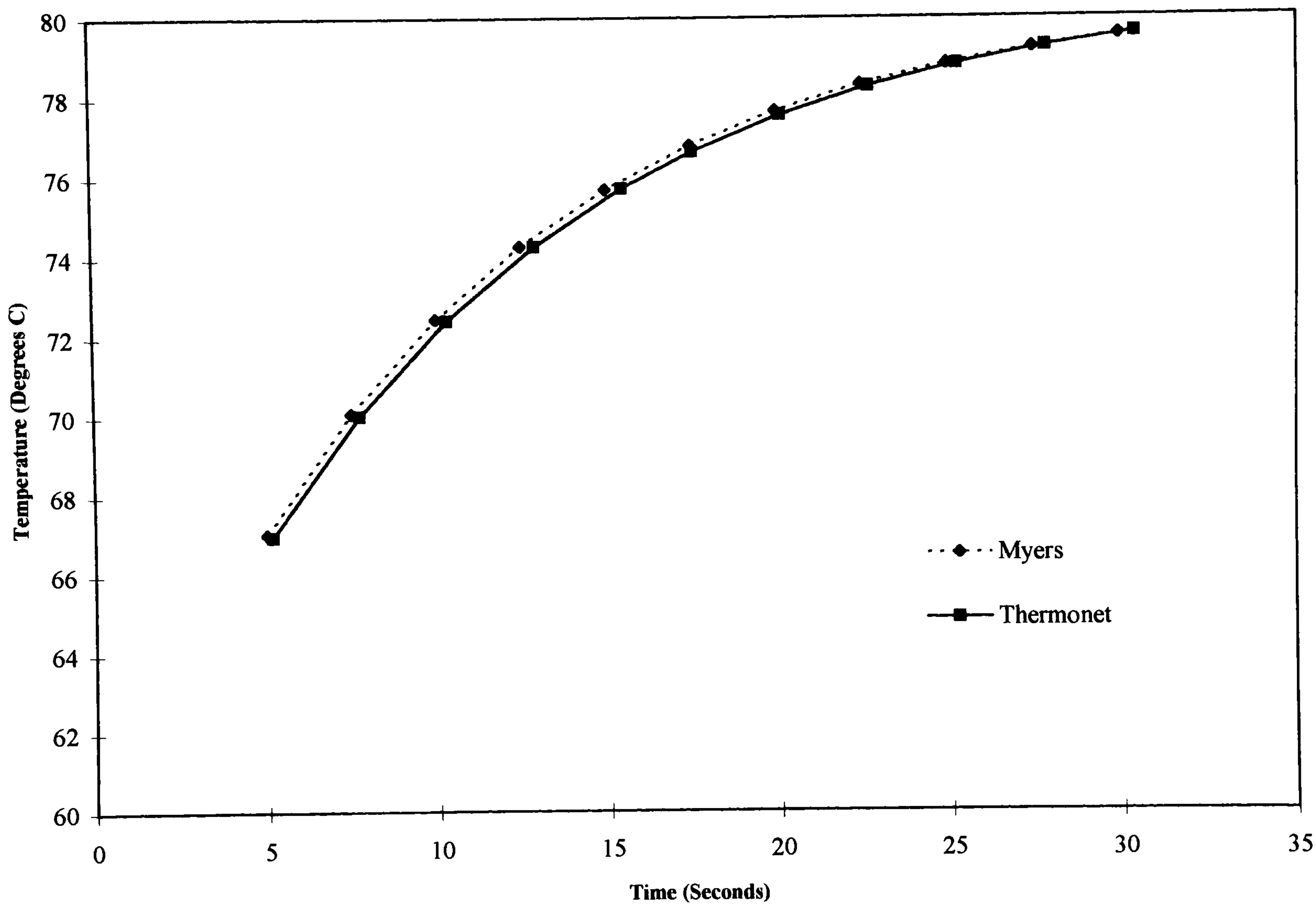


Figure B.1 Comparison of Thermonet solution to solution by Myers et al. (1970)

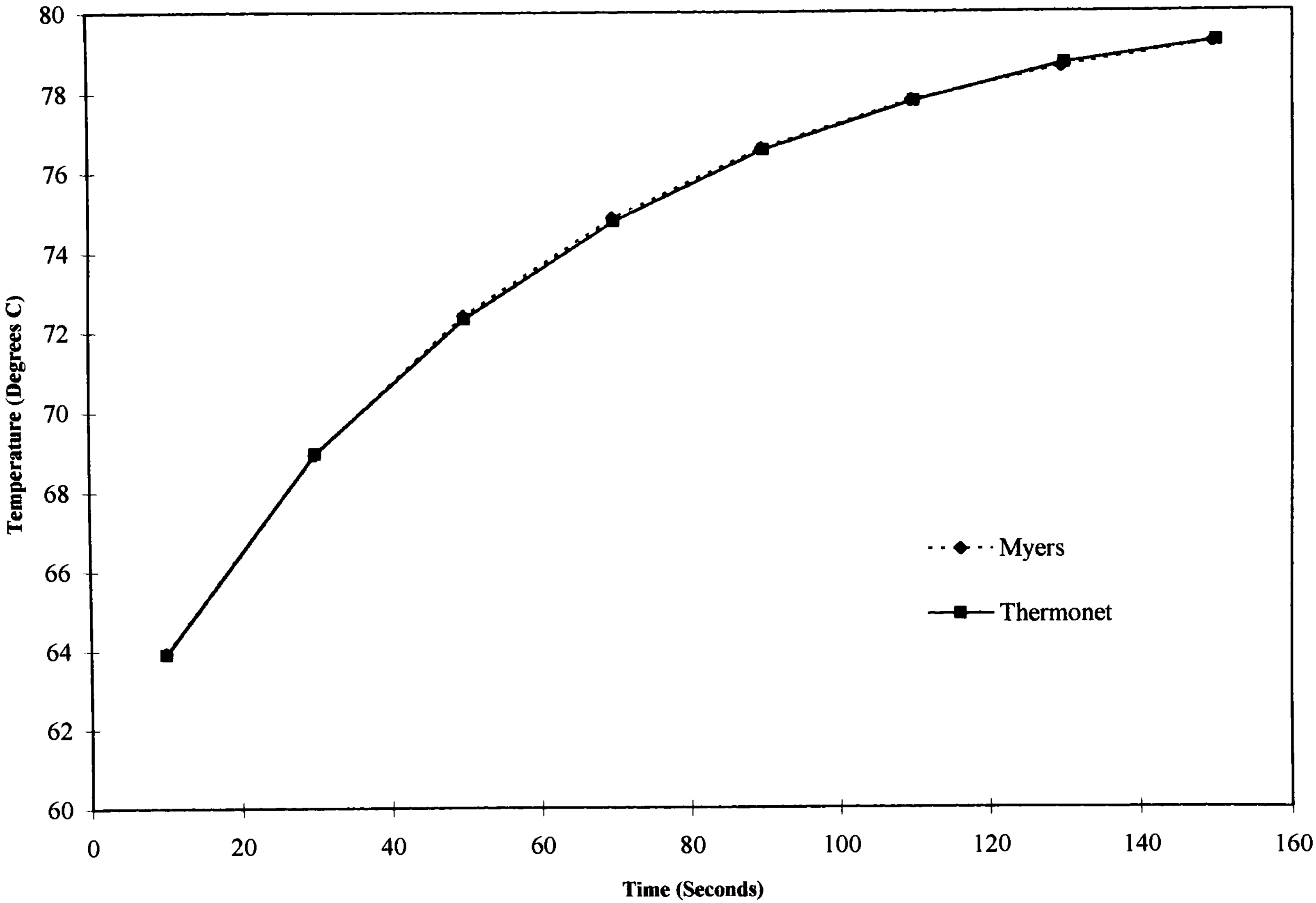


Figure B.2 Comparison of Thermonet solution to solution by Myers et al. (1970)

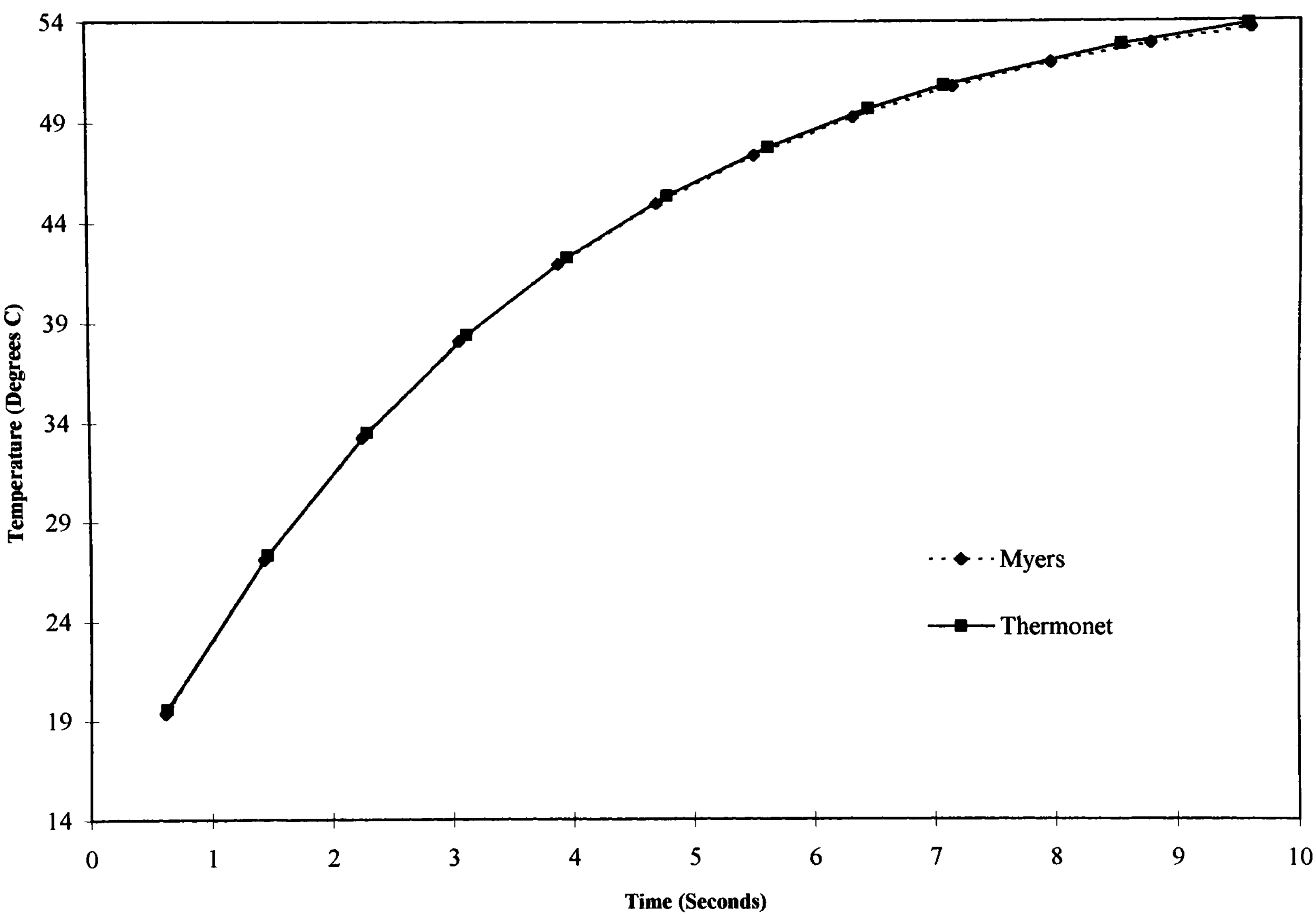


Figure B.3 Comparison of Thermonet solution to solution by Myers et al. (1970)

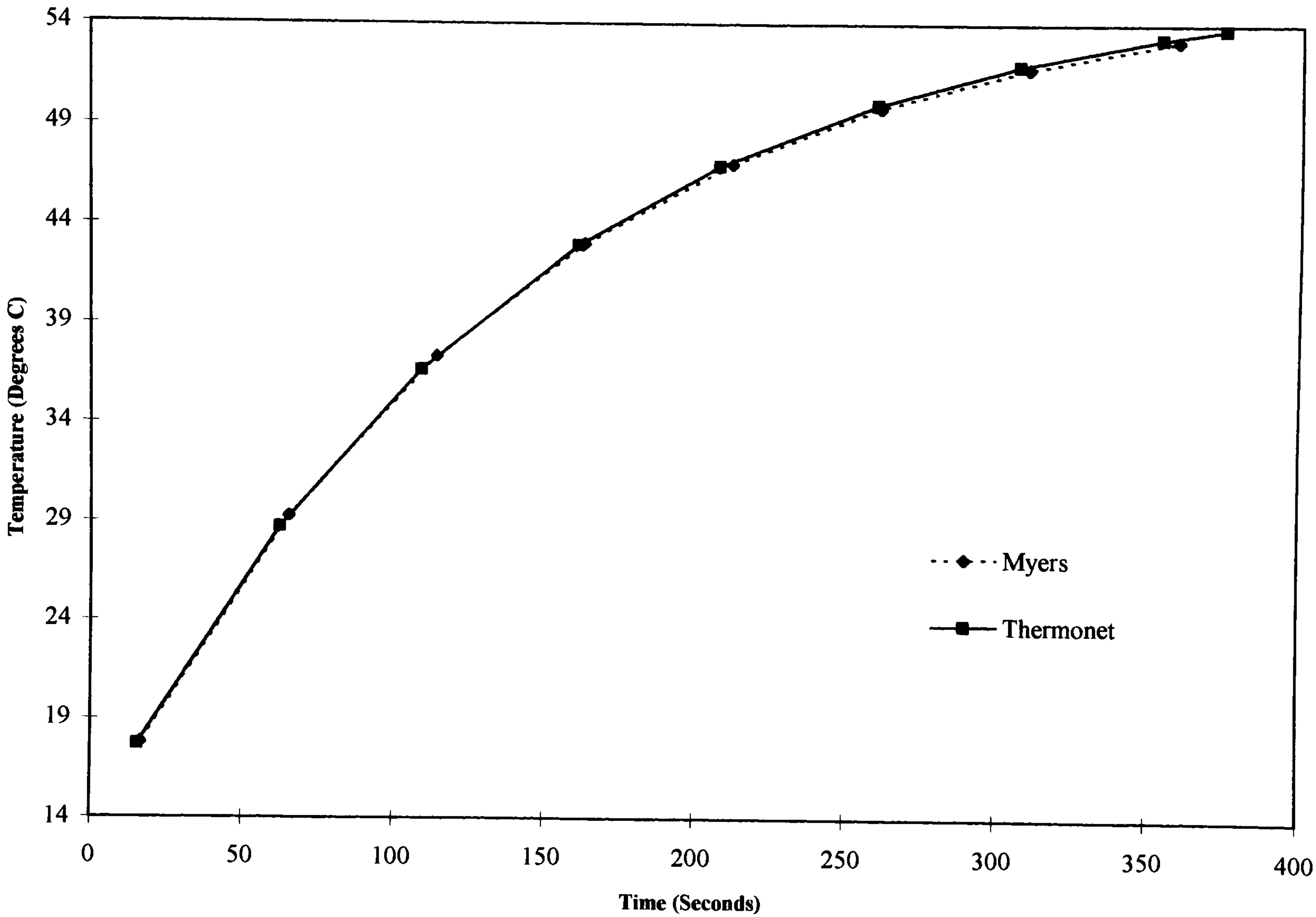


Figure B.4 Comparison of Thermonet solution to solution by Myers et al. (1970)

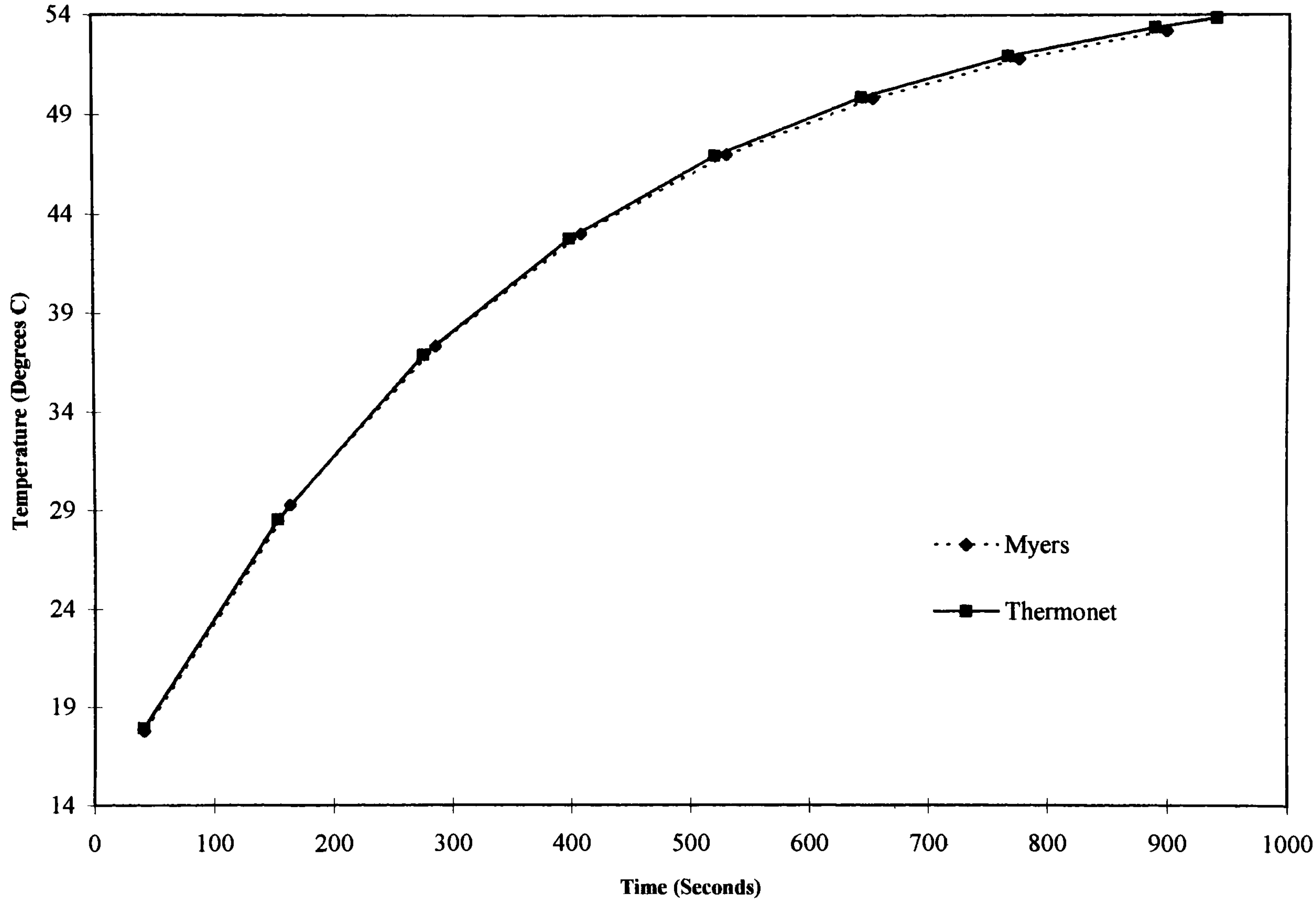


Figure B.5 Comparison of Thermonet solution to solution by Myers et al. (1970)

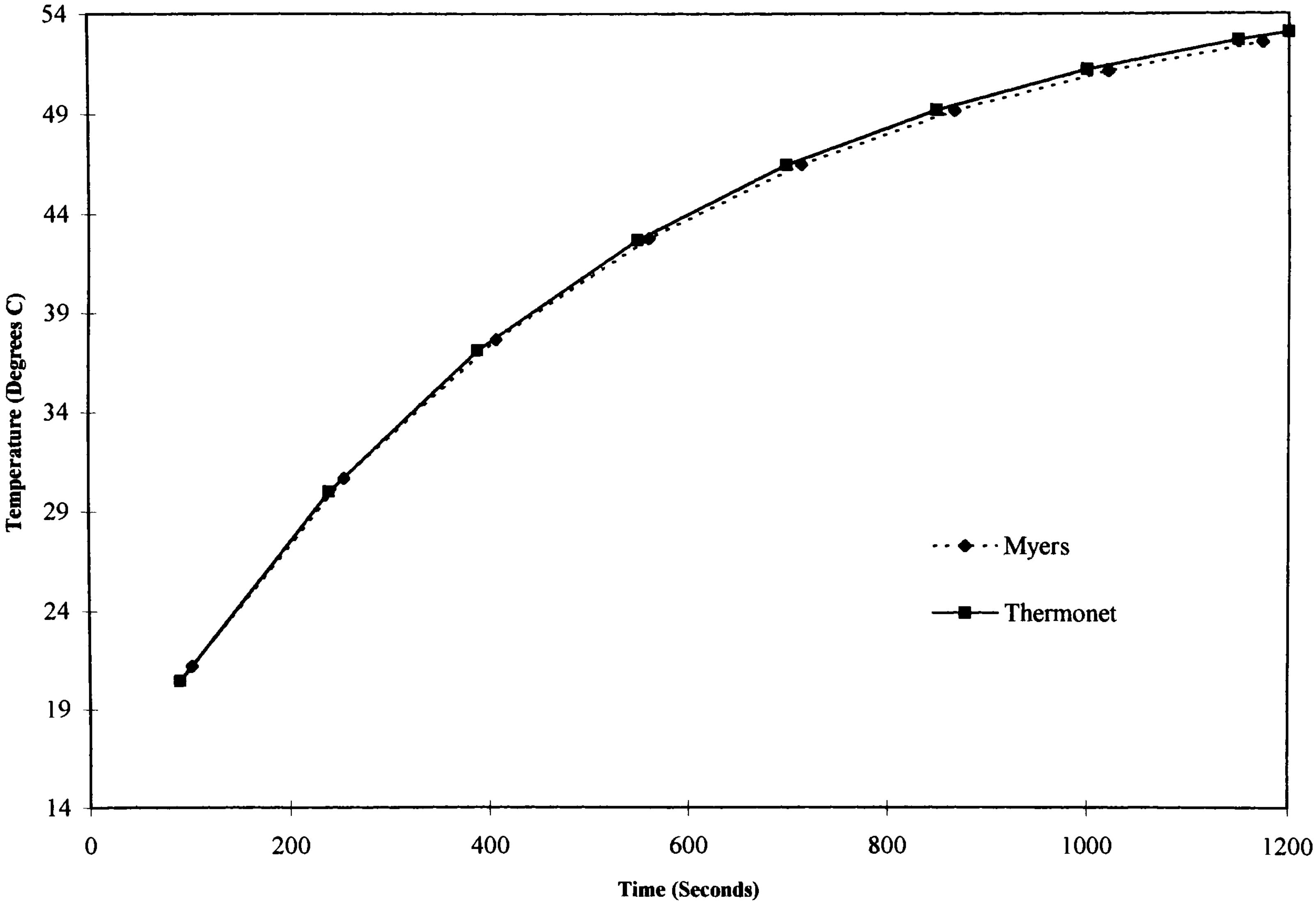


Figure B.6 Comparison of Thermonet solution to solution by Myers et al. (1970)

Appendix C

In order to determine the influence of t_d* on transient solutions, Thermonet was utilized to generate solutions that could be compared against each other. For each set of parameters listed in table C.1, two solutions were generated, one solution for $t_d^* = 0.2$ and one solution for $t_d^* = 4.0$. The two results are then compared to determine the influence of t_d* on the solution. The first task was to determine the values of NTU, C*, and R* which yield the highest deviation. For $\overline{C}_w^* = 50$ and NTU = .5 this was seen to be C* = 1.0 and R* = 0.5. Once the values of C* and R* which produce the largest deviation were determined, the influence of NTU was then investigated. It can be seen in table C.1 that a value of NTU = 5.0 did not increase the amount of deviation. It can then be concluded that the parameter values that produce the largest deviation are NTU = 0.5, C* = 1.0, and R* = 0.5. These parameter values are expected to produce the largest deviation for $\overline{C}_w^*$ = 10 as well.

Table C.1 Summary of influence of t_d* on transient solutions.

$\overline{C}_w^* = 50.0$				
NTU	C*	R*	Max. % difference	Table #
0.5	0.25	0.5	1.21 %	C.2
0.5	0.25	2.0	1.21 %	C.3
0.5	1.0	0.5	2.32 %	C.4
0.5	1.0	2.0	2.26 %	C.5
5.0	1.0	0.5	2.02 %	C.6
$\overline{C}_w^* = 10.0$				
NTU	C*	R*	Max % difference	Table #
0.5	1.0	0.5	8.60 %	C.7

Table C.2

$C_w^* = 50$ NTU = 0.5 $C^* = 0.25$ $R^* = 0.5$						
	$t_d^* = 0.2$		$t_d^* = 4.0$		% difference	
time	stepped	unstepped	stepped	unstepped	stepped	unstepped
0	51.1323	24.7169	51.1323	24.7169	0.000	0.000
6.25	70.7555	25.5762	70.7595	25.8866	0.006	1.214
12.5	74.9829	26.6139	74.9918	26.9130	0.012	1.124
18.75	78.4263	27.4949	78.4388	27.7504	0.016	0.929
25	81.3094	28.2172	81.3241	28.4326	0.018	0.763
31.25	83.7125	28.8068	83.7285	28.9890	0.019	0.632
37.5	85.7073	29.2872	85.7240	29.4416	0.019	0.527
43.75	87.3573	29.6778	87.3740	29.8048	0.019	0.428
50	88.7175	29.9947	88.7338	30.0999	0.018	0.351
56.25	89.8355	30.2517	89.8510	30.3397	0.017	0.291
62.5	90.7522	30.4597	90.7666	30.5319	0.016	0.237
68.75	91.5017	30.6278	91.5149	30.6856	0.014	0.189
75	92.1135	30.7635	92.1255	30.8126	0.013	0.160
81.25	92.6116	30.8728	92.6224	30.9119	0.012	0.127
87.5	93.0165	30.9609	93.0261	30.9944	0.010	0.108
93.75	93.3451	31.0318	93.3534	31.0572	0.009	0.082
100	93.6112	31.0887	93.6186	31.1113	0.008	0.073
106.25	93.8266	31.1344	93.8330	31.1515	0.007	0.055
112.5	94.0005	31.1711	94.0058	31.1844	0.006	0.043
118.75	94.1408	31.2005	94.1455	31.2121	0.005	0.037
125	94.2538	31.2240	94.2577	31.2319	0.004	0.025
131.25	94.3449	31.2429	94.3480	31.2488	0.003	0.019
137.5	94.4182	31.2580	94.4206	31.2624	0.003	0.014
143.75	94.4769	31.2700	94.4791	31.2730	0.002	0.010
150	94.5240	31.2796	94.5256	31.2830	0.002	0.011

Table C.3

$C_w^* = 50$ NTU = 0.5 $C^* = 0.25$ R = 2.0						
time	$t_d^* = 0.2$		$t_d^* = 4.0$		% difference	
	stepped	unstepped	stepped	unstepped	stepped	unstepped
0	51.1323	24.7169	51.1323	24.7169	0.000	0.000
3.125	70.7555	25.5762	70.7595	25.8866	0.006	1.214
6.25	74.9829	26.6139	74.9918	26.9130	0.012	1.124
9.375	78.4263	27.4949	78.4388	27.7504	0.016	0.929
12.5	81.3094	28.2172	81.3241	28.4326	0.018	0.763
15.625	83.7125	28.8068	83.7285	28.9890	0.019	0.632
18.75	85.7073	29.2872	85.7240	29.4416	0.019	0.527
21.875	87.3573	29.6778	87.3740	29.8048	0.019	0.428
25	88.7175	29.9947	88.7338	30.0999	0.018	0.351
28.125	89.8355	30.2517	89.8510	30.3397	0.017	0.291
31.25	90.7522	30.4597	90.7666	30.5319	0.016	0.237
34.375	91.5017	30.6278	91.5149	30.6856	0.014	0.189
37.5	92.1135	30.7635	92.1255	30.8126	0.013	0.160
40.625	92.6116	30.8728	92.6224	30.9119	0.012	0.127
43.75	93.0165	30.9609	93.0261	30.9944	0.010	0.108
46.875	93.3451	31.0318	93.3534	31.0572	0.009	0.082
50	93.6112	31.0887	93.6186	31.1113	0.008	0.073
53.125	93.8266	31.1344	93.8330	31.1515	0.007	0.055
56.25	94.0005	31.1711	94.0058	31.1844	0.006	0.043
59.375	94.1408	31.2005	94.1455	31.2121	0.005	0.037
62.5	94.2538	31.2240	94.2577	31.2319	0.004	0.025
65.625	94.3449	31.2429	94.3480	31.2488	0.003	0.019
68.75	94.4182	31.2580	94.4206	31.2624	0.003	0.014
71.875	94.4769	31.2700	94.4791	31.2730	0.002	0.010
75	94.5240	31.2796	94.5256	31.2830	0.002	0.011

Table C.4

$C_w^* = 50$ NTU = 0.5 $C^* = 1.0$ $R^* = 0.5$						
	$t_d^* = .2$		$t_d^* = 4$		% difference	
time	stepped	unstepped	stepped	unstepped	stepped	unstepped
0	53.3504	36.6496	53.3504	36.6496	0.000	0.000
6.25	73.0031	39.6188	73.0154	40.5394	0.017	2.324
12.5	77.3195	43.0672	77.3472	43.9627	0.036	2.079
18.75	80.8997	45.9995	80.9395	46.7863	0.049	1.711
25	83.9529	48.4296	84.0013	49.1143	0.058	1.414
31.25	86.5469	50.4403	86.6010	51.0329	0.063	1.175
37.5	88.7434	52.1029	88.8007	52.6137	0.065	0.980
43.75	90.5980	53.4771	90.6563	53.9158	0.064	0.820
50	92.1589	54.6124	92.2177	54.9879	0.064	0.688
56.25	93.4720	55.5501	93.5282	55.8705	0.060	0.577
62.5	94.5718	56.3242	94.6258	56.5970	0.057	0.484
68.75	95.4931	56.9628	95.5434	57.1948	0.053	0.407
75	96.2626	57.4906	96.3092	57.6866	0.048	0.341
81.25	96.9047	57.9252	96.9473	58.0910	0.044	0.286
87.5	97.4395	58.2834	97.4782	58.4236	0.040	0.241
93.75	97.8847	58.5798	97.9195	58.6970	0.036	0.200
100	98.2548	58.8240	98.2858	58.9219	0.032	0.166
106.25	98.5623	59.0250	98.5896	59.1066	0.028	0.138
112.5	98.8174	59.1904	98.8413	59.2585	0.024	0.115
118.75	99.0289	59.3267	99.0497	59.3832	0.021	0.095
125	99.2042	59.4392	99.2220	59.4857	0.018	0.078
131.25	99.3493	59.5321	99.3645	59.5700	0.015	0.064
137.5	99.4693	59.6085	99.4821	59.6391	0.013	0.051
143.75	99.5686	59.6713	99.5793	59.6959	0.011	0.041
150	99.6507	59.7232	99.6594	59.7426	0.009	0.032

Table C.5

$C_w^* = 50$ NTU = 0.5 $C^* = 1.0$ $R^* = 2.0$						
	$t_d^* = 0.2$		$t_d = 4.0$		% difference	
time	stepped	unstepped	stepped	unstepped	stepped	unstepped
0	53.3504	36.6496	53.3504	36.6496	0.000	0.000
3.125	88.1830	39.6400	88.2064	40.5367	0.027	2.262
6.25	90.5454	43.0852	90.5930	43.9618	0.053	2.035
9.375	92.2231	46.0098	92.2858	46.7867	0.068	1.689
12.5	93.6046	48.4342	93.6752	49.1155	0.075	1.407
15.625	94.7426	50.4406	94.8159	51.0346	0.077	1.178
18.75	95.6796	52.1002	95.7523	52.6157	0.076	0.989
21.875	96.4516	53.4725	96.5210	53.9179	0.072	0.833
25	97.0873	54.6067	97.1521	54.9901	0.067	0.702
28.125	97.6104	55.5438	97.6700	55.8726	0.061	0.592
31.25	98.0412	56.3177	98.0952	56.5990	0.055	0.500
34.375	98.3958	56.9567	98.4441	57.1966	0.049	0.421
37.5	98.6879	57.4841	98.7305	57.6882	0.043	0.355
40.625	98.9282	57.9194	98.9656	58.0926	0.038	0.299
43.75	99.1257	58.2784	99.1585	58.4251	0.033	0.252
46.875	99.2889	58.5746	99.3168	58.6979	0.028	0.211
50	99.4228	58.8188	99.4468	58.9231	0.024	0.177
53.125	99.5332	59.0201	99.5534	59.1077	0.020	0.148
56.25	99.6238	59.1860	99.6409	59.2595	0.017	0.124
59.375	99.6985	59.3228	99.7127	59.3842	0.014	0.104
62.5	99.7598	59.4355	99.7716	59.4867	0.012	0.086
65.625	99.8105	59.5283	99.8200	59.5708	0.010	0.071
68.75	99.8521	59.6048	99.8597	59.6399	0.008	0.059
71.875	99.8863	59.6678	99.8922	59.6968	0.006	0.049
75	99.9145	59.7197	99.9190	59.7434	0.004	0.040

Table C.6

$C_w^* = 50$ NTU = 5.0 $C^* = 1.0$ $R^* = 0.5$						
	$t_d^* = 0.2$		$t_d^* = 4.0$		% difference	
time	stepped	unstepped	stepped	unstepped	stepped	unstepped
0.00	28.63588	61.36412	28.63588	61.36412	0	0
10.23	28.64532	72.26808	28.64593	73.72906	0.002129	2.021605
20.45	28.67432	80.99748	28.67691	82.51867	0.009025	1.878076
30.68	28.73314	87.39566	28.73987	88.8297	0.023419	1.640856
51.14	28.97003	95.81677	28.99367	97.07901	0.081626	1.317353
71.59	29.38729	101.0106	29.441	102.1396	0.182783	1.117717
92.05	29.97384	104.522	30.06866	105.5459	0.316342	0.979554
102.27	30.3189	105.887	30.437	106.8662	0.389529	0.924819
112.50	30.69012	107.0652	30.83257	108.0043	0.464143	0.877081
122.73	31.08154	108.0951	31.24836	108.9969	0.536708	0.834289
132.95	31.48703	109.0038	31.67787	109.8709	0.606078	0.795508
153.41	32.31768	110.5367	32.55296	111.3415	0.72801	0.728092
163.64	32.73327	111.1898	32.98876	111.9652	0.780506	0.697446
184.09	33.54644	112.3206	33.83489	113.0443	0.859847	0.644277
194.32	33.93837	112.8131	34.24001	113.5119	0.888796	0.619451
214.77	34.68335	113.6797	35.00493	114.3309	0.927197	0.572829
225.00	35.03454	114.0633	35.36245	114.6906	0.935976	0.549894
245.45	35.68954	114.7439	36.02517	115.3266	0.940413	0.507795
255.68	35.99349	115.0471	36.33029	115.6083	0.935736	0.487783
276.14	36.55475	115.59	36.88936	116.1095	0.915356	0.449421
286.36	36.81273	115.8331	37.14426	116.3326	0.900569	0.431214
306.82	37.28521	116.2702	37.60799	116.7311	0.865686	0.39641
317.05	37.50083	116.4665	37.81821	116.9092	0.846333	0.380105
337.50	37.8954	116.8209	38.19876	117.2273	0.80053	0.347905
347.73	38.07455	116.9804	38.37067	117.3701	0.777745	0.333141
368.18	38.40152	117.2675	38.68104	117.6258	0.727872	0.305618
378.41	38.5492	117.3976	38.82086	117.7395	0.704699	0.291229
398.86	38.81852	117.6328	39.07285	117.945	0.655177	0.265463
409.09	38.94019	117.7389	39.18626	118.0374	0.63192	0.253534
429.55	39.16145	117.9307	39.39045	118.2024	0.584751	0.230388
450.00	39.35561	118.0984	39.5676	118.3466	0.538653	0.210137

Table C.7

$C_w^* = 10$ NTU = 0.5 $C^* = 1.0$ $R^* = 0.5$						
	$t_d^* = 0.2$		$t_d^* = 4.0$		% difference	
time	stepped	unstepped	stepped	unstepped	stepped	unstepped
0	53.3504	36.6496	53.3504	36.6496	0.000	0.000
1.5625	69.1950	38.5725	69.2212	41.1956	0.038	6.800
3.125	78.0114	41.7899	78.1192	45.7204	0.138	9.405
4.6875	82.9367	45.1666	83.1379	49.2029	0.243	8.937
6.25	86.7139	48.2135	86.9874	51.8389	0.315	7.519
7.8125	89.6797	50.7628	89.9975	53.8342	0.354	6.051
9.375	92.0024	52.8115	92.3390	55.3433	0.366	4.794
10.9375	93.8157	54.4250	94.1512	56.4820	0.358	3.779
12.5	95.2271	55.6835	95.5472	57.3393	0.336	2.973
14.0625	96.3231	56.6606	96.6199	57.9890	0.308	2.344
15.625	97.1723	57.4171	97.4409	58.4759	0.276	1.844
17.1875	97.8296	58.0018	98.0676	58.8434	0.243	1.451
18.75	98.3374	58.4533	98.5444	59.1195	0.211	1.140
20.3125	98.7292	58.8016	98.9074	59.3285	0.180	0.896
21.875	99.0313	59.0699	99.1830	59.4860	0.153	0.704
23.4375	99.2640	59.2765	99.3921	59.6040	0.129	0.553
25	99.4431	59.4355	99.5504	59.6918	0.108	0.431